CEFET/RJ
DIRETORIA DE PESQUISA E PÓS-GRADUAÇÃO

PERTURBAÇÕES EM GRAFOS E SEUS EFEITOS SOBRE A CONECTIVIDADE ALGÉBRICA

Márcia Bengio de Albuquerque

Dissertação de Mestrado apresentada ao Programa de Pós-graduação em Tecnologia do Centro Federal de Educação Tecnológica Celso Suckow da Fonseca, CEFET/RJ, como parte dos requisitos necessários para obtenção do título de Mestre em Tecnologia.

Orientadores:
Carla Silva Oliveira, D.Sc.
Leonardo Silva de Lima, D.Sc.

Rio de Janeiro
Agosto de 2012

PERTURBAÇÕES EM GRAFOS E SEUS EFEITOS SOBRE A CONECTIVIDADE ALGÉBRICA

Dissertação de Mestrado apresentada ao Programa de Pós-graduação em Tecnologia do Centro Federal de Educação Tecnológica Celso Suckow da Fonseca, CEFET/RJ, como parte dos requisitos necessários para obtenção do título de Mestre em Tecnologia.

Márcia Bengio de Albuquerque

Aprovada por:

_____

Presidente, Prof. Carla Silva Oliveira, D.Sc. (ENCE/IBGE) (Orientadora)

_____

Prof. Leonardo Silva de Lima, D.Sc. (Coorientador)

_____

Prof. Reinaldo Castro Souza, D.Sc. (PUC-Rio)

_____

Prof. Claudia Marcela Justel, D.Sc. (IME)

_____

Prof. Eduardo Bezerra da Silva, D.Sc.

Rio de Janeiro

Agosto de 2012

Ficha catalográfica elaborada pela Biblioteca Central do CEFET/RJ

A345 Albuquerque, Márcia Bengio de
Perturbações em grafos e seus efeitos sobre a conectividade algébrica / Márcia Bengio de Albuquerque, - 2012.
viii, 108f.:il., tabs.; enc.

Dissertação (Mestrado) Centro Federal de Educação Tecnológica Celso Suckow da Fonseca, 2012.
Bibliografia:f. 39-41
Orientadores:Carla Silva Oliveira [e] Leonardo Silva de Lima.
Inclui Apêndices

1.Teoria dos grafos 2.Conectividade algébrica 3.Grafos-Função de perturbação-Avaliação I.Oliveira,Carla Silva(orient.) II.Lima,Leonardo Silva de(orient.) III.Título.

CDD 511.5

RESUMO

PERTURBAÇÕES EM GRAFOS E SEUS EFEITOS SOBRE A CONECTIVIDADE ALGÉBRICA

Márcia Bengio de Albuquerque

Orientadores: Carla Silva Oliveira, D.Sc.

Leonardo Silva de Lima, D.Sc.

Resumo da Dissertação de Mestrado submetida ao Programa de Pós-Graduação em Tecnologia do Centro Federal de Educação Tecnológica Celso Suckow da Fonseca, CEFET/RJ, como parte dos requisistos à obtenção do título de Mestre em Tecnologia.

Neste trabalho, determinamos as variações sofridas pela conectividade algébrica de um grafo, um parâmetro espectral, denotado por $a(G)$, ao aplicarmos a operação de remoção de um vértice nos grafos das famílias *split* completo, abacaxi, caminho completo, dupla estrela e em grafos resultantes das operações $1-$soma, $2-$soma e ponte de ligação entre grafos completos. A função utilizada para medir esta variação é dada por $\phi(v) = a(G) - a(G \setminus v)$, onde $G \setminus v$ é o grafo obtido de G pela remoção do vértice v e de todas as suas arestas incidentes. Um artigo recente de autoria de Kirkland estabeleceu condições necessárias e suficientes para G tal que $\phi(v) = 1$ e, como consequência desse resultado, mostrou que, se G tem um vértice universal v e este vértice é removido, tem-se $\phi(v) = 1$. A partir desse fato, o autor levantou a seguinte questão: "O que podemos dizer sobre $\phi(v)$ se o grau do vértice v é $(n-2)$?" Nesse estudo, resolvemos parcialmente esta questão e obtivemos condições necessárias para um grafo G tal que $\phi(v) = 0$ quando v tem grau $(n-2)$. Estabelecemos ainda duas conjecturas acerca da função $\phi(v)$ quando v tem grau $(n-2)$.

Palavras-chave:

Grafos; Conectividade Algébrica; Perturbações.

Rio de Janeiro

Agosto / 2012

ABSTRACT

DISTURBANCES IN GRAPHS AND ITS EFFECTS ON ALGEBRAIC CONNECTIVITY

Márcia Bengio de Albuquerque

Advisor(s): Carla Silva Oliveira, D.Sc.

Leonardo Silva de Lima, D.Sc.

Abstract of dissertation submitted to Programa de Pós-Graduação em Tecnologia do Centro Federal de Educação Tecnológica Celso Suckow da Fonseca, CEFET/RJ, as partial fulfillment of the requirements for the degree of Technology Master.

In this work, we consider the operation of a vertex removal and its effects to the algebraic connectivity of a graph, that is a spectral parameter denoted by $a(G)$. That operation is applied to some families of graphs, such that complete split, pineapple, path-complete, double star, $1-$sum and $2-$sum graphs, and linking bridge between complete graphs. We used the function $\phi(v) = a(G) - a(G \setminus v)$ to evaluate the algebraic connectivity variation when $G \setminus v$ is the graph obtained from G by removing the vertex v and all of its incident edges. A recent paper authored by Kirkland establishes necessary and sufficient conditions for G such that $\phi(v) = 1$ and as a consequence of that result, he also shows that if G has a universal vertex v, then $\phi(v) = 1$. From this fact, Kirkland has risen the question: "What can we say about $\phi(v)$ if vertex degree of v is $(n-2)$?" Here, we partially solve this problem obtaning a necessary condition to a graph G such that $\phi(v) = 0$ when v is of degree $(n-2)$. Also , we state two conjectures to $\phi(v)$ when v is of degree $(n-2)$.

Keywords:

Graphs; Algebraic Connectivity; Disturbances.

Rio de Janeiro

August / 2012

Sumário

Introdução **1**

I Conceitos preliminares **4**

II Remoção de um vértice em famílias de grafos e a conectividade algébrica **13**

II.1 Função de perturbação: resultados gerais 13

II.2 Função de perturbação: resultados em algumas famílias de grafos 15

III Função de perturbação em grafos que possuem pelo menos um vértice de grau $(n-2)$ **25**

III.1 *Split* Completo, Caminho Completo, Dupla estrela 25

III.2 Grafos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$ 26

IV Grafos com função de perturbação nula que possuem pelo menos um vértice de grau $(n-2)$ **28**

IV.1 Grafos caminho completo 28

IV.2 Grafos gerais 29

IV.3 Resultados Teóricos 33

Conclusões e Trabalhos Futuros **38**

Referências Bibliográficas **39**

A Grafos que possuem vértices de grau $(n-2)$ e seus subgrafos **42**

B Análise da função de perturbação para grafos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$, que possuem vértices de grau $(n-2)$ **80**

Lista de Figuras

FIG. I.1 Exemplos de grafos *split* 5

FIG. I.2 SC(7,3), SC(6,2) e SC(7,4) 5

FIG. I.3 PA(7,4), PA(9,5) e PA(11,6) 6

FIG. I.4 Um grafo *path-complete* (n, p, t) 6

FIG. I.5 **(a)** C_4 e $K_{1,4}$; **(b)** Grafo $1-$soma entre C_4 e $K_{1,4}$ $(C_4 \circ K_{1,4})$ 7

FIG. I.6 **(a)** K_3 e $SC(6,3)$; **(b)** Grafo $2-$soma entre K_3 e $SC(6,3)$ $(K_3 \bullet SC(6,3))$ . . . 7

FIG. I.7 **(a)** P_3 e G_2; **(b)** Grafo ponte de ligação entre P_3v_1 e G_2v_2 $(P_3v_1 : G_2v_2)$ 7

FIG. I.8 $D_{p,q}$ 8

FIG. I.9 Um grafo G 11

FIG. II.1 $PA(n, q)$ e o seu complementar 16

FIG. II.2 $PC_{n,p,1}$ e seu complementar 17

FIG. III.1 $D_{1,q}$, com $q \geq 3$ 26

FIG. IV.1 Estrutura de um grafo $G \in \Im_{n,n-2}$ 34

Lista de Tabelas

TAB. I.1 Alguns grafos para os quais $a(G)$ é conhecida 10

TAB. II.1 Resultados obtidos para $\phi(v)$ 23

TAB. III.1 Resultados para $\phi(v)$ 26

TAB. IV.1 Grafos com 4 vértices 30
TAB. IV.2 Grafos com 5 vértices 30
TAB. IV.3 Grafos com 6 vértices 30
TAB. IV.4 Grafos com 7 vértices 31
TAB. IV.5 Outros resultados para $\phi(v)$ 37

TAB. B.1 Grafos com 4 vértices 80
TAB. B.2 Grafos com 5 vértices 80
TAB. B.3 Grafos com 6 vértices 81
TAB. B.4 Grafos com 7 vértices 85

Introdução

A Teoria Espectral dos Grafos (TEG) teve origem na Química Quântica quando Huckel [24], em 1931, produziu um modelo teórico para um dado problema em que os níveis de energia de certos elétrons eram representados por autovalores de um grafo, a partir de moléculas de hidrocarbonetos não saturadas. No entanto, somente a partir da tese de doutorado de Cvetković [6], em 1971, que esta teoria teve sua fundamentação teórica sedimentada. O crescimento desta teoria se deu naturalmente em diversas áreas, como por exemplo, na Física Teórica e Mecânica Quântica, na tentativa de minimizar a energia de sistemas Hamiltonianos. Uma das primeiras motivações do estudo em Teoria Espectral dos Grafos foi a esperança de se caracterizar grafos por meio dos seus espectros. Mesmo concluindo que essa conjectura não era verdadeira a pesquisa não se diluiu, ao contrário, ganhou muitos adeptos. Em 1980, os resultados relacionados à Teoria Espectral dos Grafos publicados até 1978 foram reunidos em [8]. Em 1988, esta coletânea foi complementada por Cvetković, Doob, Gutman e Torgasev em [10]. Os livros An Introduction to the Theory of Graph Spectra [11], Graph Spectra of Graphs [1], Matemática Discreta: Grafos, Redes, Aplicações [18] são os mais recentes publicados nesta área. Um dos principais objetivos da Teoria Espectral dos Grafos é deduzir propriedades em grafos através de informações espectrais fornecidas por matrizes associadas a grafos, como por exemplo, a matriz de adjacência, laplaciana e laplaciana sem sinal.

A matriz laplaciana de um grafo G, L(G), é uma matriz simétrica e semidefinida positiva, que tem o zero como um dos seus autovalores. No caso em que G é conexo, então zero é um autovalor simples e a multiplicidade do zero coincide com o número de componentes conexas de G, [30]. A conectividade algébrica, segundo menor autovalor da matriz laplaciana, é um autovalor relacionado à conexidade do grafo, que desempenha um papel importante na Teoria Espectral dos Grafos. Por isso, há uma riqueza de resultados envolvendo este parâmetro começando com o trabalho de Fiedler [22]. Os surveys [25] e [32] fornecem visões gerais sobre este parâmetro e sobre os vetores de Fiedler. Existem trabalhos que analisam o comportamento da conectividade algébrica a partir de perturbações em grafos obtidas pela remoção de vértices e arestas, vide [21], [2], [28], [34], [35] e [37]. A conectividade algébrica está associada a outros invariantes importantes, como por exemplo, conectividade de vértice e de aresta, número isoperimétrico e diâmetro, corte maximal, número independente, dentre outros, como pode ser visto em [25]. Além disso, uma aplicação bastante interessante desse parâmetro é como medida de vulnerabilidade de uma rede, que é a propensão da rede se desconectar após um ataque consistindo na remoção de um

de seus vértices e/ou arestas, [23]. A vulnerabilidade de uma rede também está relacionada com a conectividade de vértices e de arestas do grafo, [3], [9], [12], [14], [15], [17], [23], [5] e [27].

Objetivos

Esta pesquisa tem como objetivo geral analisar o comportamento da função de perturbação $\phi(v)$ ao remover um vértice de grau $(n-2)$ em grafos conexos. Para atingi-lo, foram estabelecidos os seguintes objetivos específicos: (i)calcular o espectro de algumas famílias de grafos e de grafos resultantes de operações e de seus subgrafos, obtidos a partir da remoção de um vértice v; (ii)identificar nestas famílias e em todos os grafos conexos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$, quais possuem vértices de grau $(n-2)$; (iii)selecionar os grafos que geram função de pertubação nula, ao remover os vértices de grau $(n-2)$; (iv)analisar a estrutura desses grafos; (v)caracterizar a matriz laplaciana e o vetor de Fiedler.

Metodologia

A metodologia utilizada na pesquisa foi dividida da seguinte forma:(i)analisamos a função de perturbação $\phi(v)$ em algumas famílias de grafos e em grafos resultantes de operações; (ii)verificamos dentre esses grafos, quais possuíam vértices de grau $(n-2)$ e, se possuíam, que função de perturbação eles geravam; (iii)com o auxílio do AGX [16], analisamos a função de perturbação em todos os grafos conexos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$ que possuíam vértice de grau $(n-2)$; (iv)selecionamos os grafos que geravam função de perturbação nula e, com o auxílio do OCTAVE [20], analisamos os vetores de Fiedler; e, por fim, (v)analisamos a estrutura desses grafos.

Estrutura do trabalho

Esta dissertação de mestrado se desenvolve iniciando o Capítulo I com os pré-requisitos básicos da Álgebra Linear, Teoria dos Grafos e Teoria Espectral dos Grafos necessários para a compreensão deste texto. O desenvolvimento dos capítulos seguintes é baseado na avaliação de uma função de perturbação. Essa função é dada pela diferença entre a conectividade algébrica de um grafo G e a conectividade algébrica de G após a remoção de um vértice e foi definida inicialmente por Kirkland em 2010, [33]. No Capítulo II, a função é avaliada em famílias específicas de grafos tais como: *split* completo, abacaxi, caminho completo, dupla estrela e os grafos resultantes das operações $1-$soma, $2-$soma e ponte de ligação entre grafos completos. No Capítulo III, são apresentados resultados envolvendo a função $\phi(v)$ em algumas famílias de grafos estudadas no Capítulo II e em grafos com n vértices, $4 \leq n \leq 7$, que possuem pelo menos um

vértice de grau $(n-2)$. Já no Capítulo IV são apresentados os testes computacionais sobre os grafos apresentados no Capítulo III, que geram $\phi(v) = 0$ e os resultados teóricos obtidos dessa dissertação.

Capítulo I - Conceitos preliminares

A Teoria Espectral dos Grafos procura obter propriedades estruturais dos grafos a partir de propriedades dos autovalores de matrizes relacionadas a grafos com o uso de resultados advindos da Álgebra Linear. Sendo assim, reunimos neste capítulo alguns resultados da Álgebra Linear, da Teoria dos Grafos e da Teoria Espectral dos Grafos, que são necessários para a compreensão desse texto. As principais fontes bibliográficas para o desenvolvimento deste capítulo foram [4], [6] e [24].

Neste trabalho consideramos $G = (V, E)$ um grafo simples, não orientado tal que V seja o conjunto de seus vértices e E o de suas arestas, tal que $|V| = n$ e $|E| = m$, onde $m \leq \frac{n(n-1)}{2}$. Cada aresta e de G, é denotada por $\{x, y\}$ quando $x, y \in V$ e dizemos que x e y são vértices adjacentes. O grau de um vértice v, denotado por $d(v)$, é o número de vértices adjacentes a ele e o conjunto constituído por esses vértices, denotado por $N(v)$, é denominado conjunto dos vizinhos de v. O grau mínimo de um grafo G, denotado por $\delta(G)$, é o número $\delta(G) = min\{d(v)/v \in V(G)\}$. Entende-se por vértice universal, o vértice que possui grau $(n-1)$. Entende-se por grafo completo, denotado por K_n, aquele em que $|V| = n$ e $|E| = C_{n,2}$; por grafo trivial, $|V| = 1$ e $|E| = 0$ e por grafo totalmente desconexo, aquele em que $|V| = n$ e $|E| = 0$. Uma cadeia de comprimento k é uma seqüência de vértices $v_1, \ldots, v_k$ tal que $\{v_j, v_{j+1}\} \in E(G)$ $\forall \ \ 1 \leq j \leq k_{j-1}$. Quando os vértices $v_1, \ldots, v_k$ são todos distintos, essa cadeia é denominada caminho simples. O grafo que é constituído apenas por um caminho simples com k vértices é denotado por P_k. Um ciclo com k vértices é uma cadeia $v_1, \ldots, v_k v_1$ onde $v_1 \ldots v_k$ é um caminho simples com $k \geq 3$, denotado por C_k. Um grafo bipartido G é um grafo cujo conjunto de vértices V pode ser particionado em dois subconjuntos V_1 e V_2, tal que não existam arestas entre vértices da mesma partição. Dizemos que G é bipartido completo, e denotamos por $K_{r,s}$, com $r + s = n$, se G tem todas as arestas possíveis entre as partições V_1 e V_2. Sejam $G = (V, E)$ e $G' = (V', E')$ dois grafos. Se $V' \subseteq V$ e $E' \subseteq E$, então G' é um subgrafo de G, ou seja, $G' \subseteq G$. E ainda, se $G' \subseteq G$ e G' contém todas as arestas $\{x, y\} \in E$ com $x, y \in V'$, então G' é um subgrafo induzido de G, denotado por $G[V']$. Uma clique de um grafo G é um subconjunto de vértices que induz um subgrafo completo. Grafo complementar, denotado por $\overline{G}$, é aquele que possui o mesmo conjunto de vértices e as arestas não existentes em G. Um conjunto independente é um conjunto de vértices que não são adjacentes entre si. Conectividade de vértices de um grafo, denotada por $k(G)$, é o menor número de vértices que, ao serem retirados, tornam o grafo desconexo ou o grafo trivial. Conectividade de arestas, denotada por $k'(G)$, é o menor número de arestas que, ao

serem retiradas, tornam o grafo desconexo. A seguir, são apresentadas definições de algumas famílias de grafos e operações em grafos, que são utilizadas nos Capítulos II e III.

Definição I.1. [29] Um grafo G é denominado *split* se e somente se o seu conjunto de vértices pode ser particionado em um conjunto independente e uma clique.

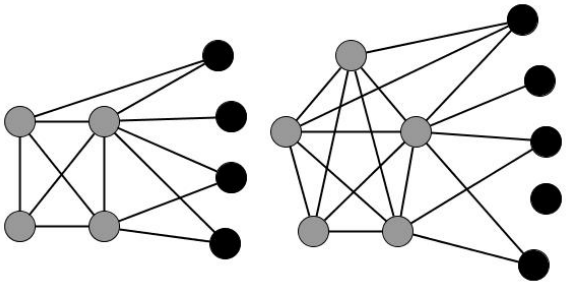


Figura I.1: Exemplos de grafos *split*

Definição I.2. [29] Sejam q e n inteiros tais que $1 \leq q \leq n$. Um grafo *split* completo, denotado por $SC(n, q)$, é um grafo com n vértices e uma clique de tamanho q, tal que o conjunto independente tem cada um de seus $n - q$ vértices adjacentes a todos os q vértices da clique.

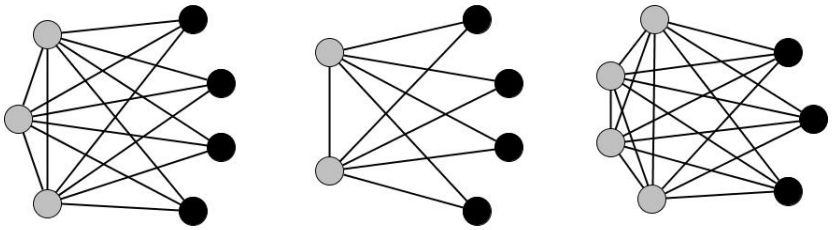


Figura I.2: SC(7,3), SC(6,2) e SC(7,4)

Definição I.3. [7] Sejam q e n inteiros tais que $1 \leq q \leq n$. Um grafo abacaxi *(pineapple graph)*, denotado por $PA(n, q)$, é aquele com n vértices consistindo de uma clique de tamanho q e um conjunto independente com $n-q$ vértices no qual cada vértice do conjunto independente é ligado ao mesmo vértice da clique.

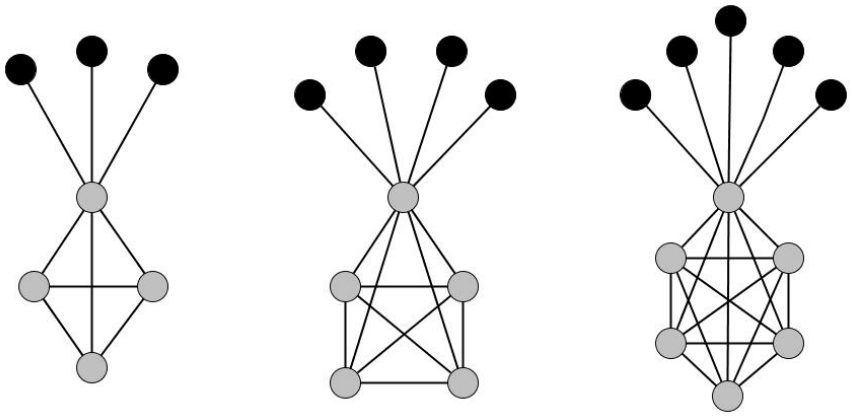


Figura I.3: PA(7,4), PA(9,5) e PA(11,6)

Definição I.4. [13] Sejam n, m, t e $p \in \mathbb{N}$, com $1 \leq t \leq n-2$ e $1 \leq p \leq n-t-1$. Um grafo com n vértices e m arestas, tal que $\frac{(n-t)(n-t-1)}{2}+t \leq m \leq \frac{(n-t)(n-t-1)}{2}+n-2$ é denominado grafo caminho completo (*path-complete*) (n,p,t), denotado $PC_{n,p,t}$, se, e somente se,

1. a clique máxima de $PC_{n,p,t}$ é K_{n-t};
2. $PC_{n,p,t}$ tem um t-caminho $P_{t+1} = [v_0, v_1, v_2, \ldots, v_t]$ tal que $v_0 \in K_{n-t} \cap P_{t+1}$ e v_1 está ligado a K_{n-t} por p arestas;
3. não há outras arestas.

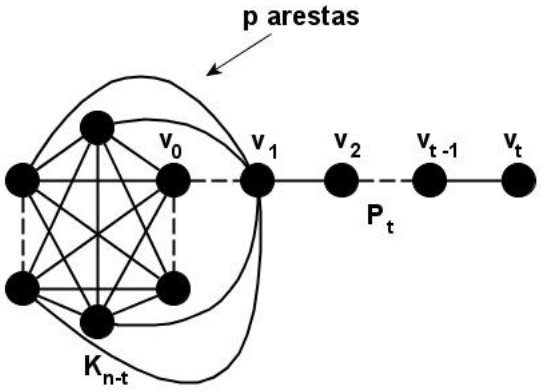


Figura I.4: Um grafo *path-complete* (n,p,t)

Definição I.5. [36] Sejam $k \geq 0$, G_1 e G_2 dois grafos disjuntos, e sejam H_1 e H_2 $k-$cliques em G_1 e G_2, respectivamente. Um grafo obtido pela identificação de H_1 e H_2 é denominado uma $k-$soma de G_1 e G_2.

Observa-se que a $0-$soma é a união de G_1 e G_2. A $1-$soma é também conhecida como coalescência de vértices entre G_1 e G_2, e a $2-$soma como coalescência de arestas entre G_1 e

G_2, as quais são denotadas por $G_1 \circ G_2$ e $G_1 \bullet G_2$, respectivamente. As Figuras I.5 e I.6 ilustram as operações da definição I.5.

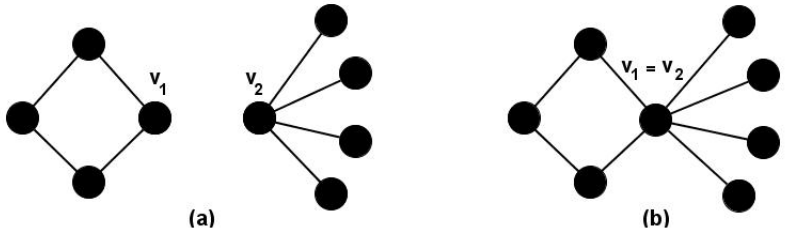


Figura I.5: **(a)** C_4 e $K_{1,4}$; **(b)** Grafo $1-$soma entre C_4 e $K_{1,4}$ $(C_4 \circ K_{1,4})$

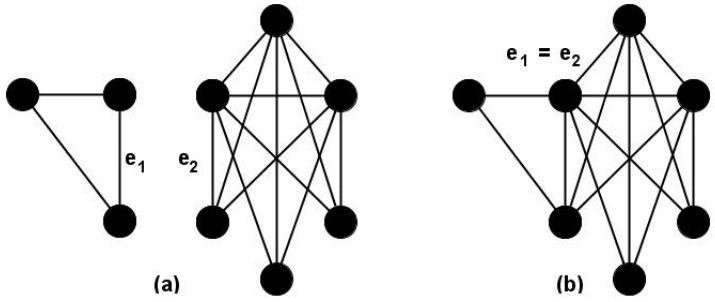


Figura I.6: **(a)** K_3 e $SC(6,3)$; **(b)** Grafo $2-$soma entre K_3 e $SC(6,3)$ $(K_3 \bullet SC(6,3))$

Definição I.6. Uma ponte de ligação entre dois grafos disjuntos G_1 e G_2, denotada por $G_1 v_1 : G_2 v_2$, é um grafo obtido a partir da ligação de um vértice $v_1 \in G_1$ ao vértice $v_2 \in G_2$ por uma aresta.

A Figura I.7 ilustra a operação da definição I.6.

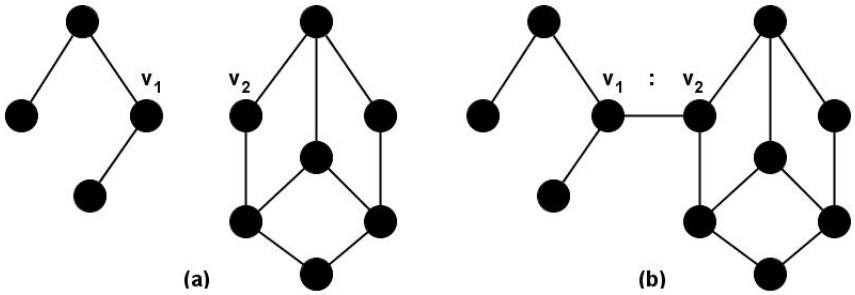


Figura I.7: **(a)** P_3 e G_2; **(b)** Grafo ponte de ligação entre $P_3 v_1$ e $G_2 v_2$ $(P_3 v_1 : G_2 v_2)$

Definição I.7. [31] Seja $\Gamma_{n,d}$ o conjunto de todas as árvores com n vértices e diâmetro d, com $2 \leq d \leq n-1$.

Obviamente, se $T \in \Gamma_{n,2}$, então T é uma estrela $(T = K_{1,n-1})$, e se $T \in \Gamma_{n,n-1}$, então T é um caminho $(T = P_n)$. Se $d = 3$, a árvore é usualmente denominada de dupla estrela [31] e denotada por $D_{p,q}$, onde p e q são os números dos vértices pendentes, como pode ser visto na Figura I.8.

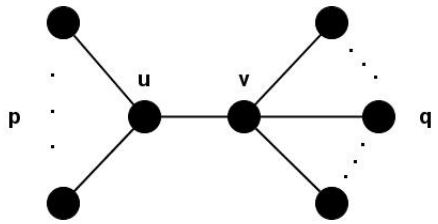


Figura I.8: $D_{p,q}$

Claramente, $D_{p,q} \in \Gamma_{p+q+2,3}$, onde $p, q \geq 1$ e $D_{p,q}$ também pode ser visto como uma ponte de ligação $K_{1,p}u : K_{1,q}v$, onde u e v são os vértices não pendentes.

Definição I.8. Sejam $G_1 = (V_1, E_1)$ e $G_2 = (V_2, E_2)$ grafos disjuntos em relação ao conjunto de vértices.

i) O grafo soma é $G_1 + G_2 = (V_1 \cup V_2, E_1 \cup E_2)$.

ii) O join $G_1 \vee G_2$, de G_1 e G_2 é um grafo obtido através de $G_1 + G_2$ adicionando todas as possíveis arestas entre os vértices de G_1 e vértices de G_2.

A seguir, apresentamos algumas definições e os principais resultados da Álgebra Linear para o desenvolvimento do tema aqui proposto.

Definição I.9. Seja A uma matriz quadrada de ordem *n*. Um autovalor de A é um escalar λ tal que $Ax = \lambda x$, para $x \neq 0$, $x \in \mathbb{R}^n$. Todo $x \in \mathbb{R}^n$ que satisfaz a esta equação é denominado um autovetor de A associado ao autovalor λ. Assim, λ é um autovalor de A, se e somente se

$$(A - \lambda I_n)x = 0$$

tem solução não trivial.

Definição I.10. Seja I_n a matriz identidade de ordem n. O polinômio de grau n associado à A, definido por

$$p_A(\lambda) = det(A - \lambda I_n)$$

é denominado polinômio característico de A.

Os autovalores de A são as raízes do polinômio característico de A e, portanto, A tem n autovalores não necessariamente distintos. O número de vezes que um autovalor é raiz do

polinômio é denominado multiplicidade algébrica. Os Teoremas I.11 e I.12 exibem propriedades das raízes do polinômio característico associadas a matrizes simétricas.

Teorema I.11. Todas as raízes do polinômio característico de uma matriz simétrica são números reais.

Definição I.12. Uma matriz A de ordem n é denominada semidefinida positiva se e somente se todos os seus autovalores são maiores ou iguais a zero.

Para finalizar este capítulo, alguns conceitos introdutórios sobre Teoria Espectral dos Grafos são apresentados.

Definição I.13. A matriz de adjacência de G, denotada por $A(G)$, é uma matriz quadrada de ordem *n* cujas entradas são

$$a_{ij} = \begin{cases} 1, & \text{se} \quad \{v_i, v_j\} \in \mathsf{E} \quad para \quad v_i, v_j \in \mathsf{V}; \\ 0, & \text{caso contrário.} \end{cases}$$

Definição I.14. A matriz laplaciana de G é uma matriz simétrica, denotada por $L(G)$, definida por

$$L(G) = D(G) - A(G),$$

onde $D(G)$ é uma matriz diagonal, cujos elementos da diagonal principal são os graus dos vértices de G e $A(G)$ a sua matriz de adjacência.

Definição I.15. O espectro da matriz laplaciana de G, denotado por $spec(L(G))$, é a sequência dos autovalores de $L(G)$ ordenados em ordem não-crescente. Assim, se $\mu_1(G) \geq \ldots \geq \mu_{n-1}(G) \geq \mu_n(G)$ são os autovalores de $L(G)$ e $m(\mu_i)$ suas respectivas multiplicidades algébricas, então

$$Spec(L(G)) = (\mu_1(G)^{m(\mu_1(G))} \quad \ldots \quad \mu_{n-1}(G)^{m(\mu_{n-1}(G))} \quad \mu_n(G)^{m(\mu_n(G))}).$$

Definição I.16. O segundo menor autovalor da matriz laplaciana de G, denominado conectividade algébrica de G é denotado por $a(G) = \mu_{n-1}(G)$ e o autovetor associado à $a(G)$ é denominado vetor de Fiedler.

A conectividade algébrica é um invariante que desempenha papel importante na Teoria Espectral de Grafos e está associado a diferentes invariantes de grafos importantes, como por exemplo, conectividade de vértice e de aresta, número isoperimétrico e diâmetro, corte maximal, número independente, dentre outros. Um survey sobre $a(G)$ pode ser encontrado em [25]. Na Tabela I.1, são apresentados os valores das conectividades algébricas para algumas famílias de

grafos [26].

Grafo	$a(G)$
K_n	$a(K_n) = n$
P_n	$a(P_n) = 2(1 - \cos(\frac{\pi}{n}))$
C_n	$a(C_n) = 2(1 - \cos(\frac{2\pi}{n}))$
$K_{p,q}$	$a(K_{p,q}) = min(p, q)$
$K_{1,n}$, com $n > 1$	$a(K_{1,n}) = 1$

Tabela I.1: Alguns grafos para os quais $a(G)$ é conhecida

As Proposições I.17 a I.20 e o Teorema I.21 apresentam limites superiores para a conectividade algébrica de um grafo e que serão úteis no desenvolvimento deste trabalho.

Proposição I.17. [22] Sejam $G_1 = (V_1, E_1)$ e $G_2 = (V_2, E_2)$ dois grafos tais que $V_1 = V_2$ e $E_1 \subseteq E_2$. Então, $a(G_1) \leq a(G_2)$.

Proposição I.18. [22] Seja G um grafo com n vértices e k vértices independentes. Então, $a(G) \leq n - k$.

Proposição I.19. [22] Se G é um grafo não-completo com n vértices, então $a(G) \leq n - 2$.

Proposição I.20. [22] Se G é um grafo não-completo, então $a(G) \leq k(G) \leq k'(G)$ e $a(G) \leq \delta(G)$.

Teorema I.21. [22] Seja $G(V, E)$ um grafo conexo $\forall$ vértice $v \in V$. Então

$$a(G) - a(G \setminus v) \leq 1,$$

onde $G \setminus v$ é o subgrafo obtido de G pela remoção do vértice v e de todas as arestas incidentes nele.

Em [30] é provado que $L(G)$ é positiva semidefinida e consequentemente seus autovalores são positivos, ou seja, $\mu_1(G) \geq \ldots \geq \mu_{n-1}(G) \geq \mu_n(G) = 0$. A Proposição I.22 relaciona o posto da matriz laplaciana com o número de componentes conexas do grafo e a Proposição I.23, a conexidade do grafo com a sua conectividade algébrica.

Proposição I.22. [29] O posto de $L(G)$ é $n - w(G)$, onde $w(G)$ é o número de componentes conexas de G.

Como consequência da proposição I.22, tem-se que $\mu_n = 0$ e, consequentemente, $Spec(L(G)) = ({\mu_1}^{m(\mu_1)} \quad \ldots \quad {\mu_{n-1}}^{m(\mu_{n-1})} \quad 0)$.

Proposição I.23. [29] Seja $Spec(L(G)) = ({\mu_1}^{m(\mu_1)} \quad \dots \quad {\mu_{n-1}}^{m(\mu_{n-1})} \quad 0)$ o espectro da matriz laplaciana de G. G é conexo se e somente se $\mu_{n-1} = a(G) \neq 0$.

O Exemplo I ilustra a conectividade algébrica de um grafo conexo e seu correspondente autovetor de Fiedler.

Exemplo I:

Seja G o grafo da Figura I.9.

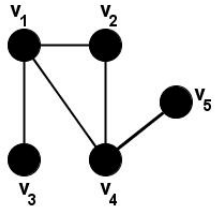


Figura I.9: Um grafo G

Para determinarmos a matriz laplaciana de G, precisamos calcular $D(G)$ e $A(G)$. Assim, temos:

$$D(G) = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad e \quad A(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Logo, a matriz laplaciana de G é

$$L(G) = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

Então, temos $Spec(L(G)) = (4.30\ 3.62\ 1.38\ 0.70\ 0)$, $a(G) = 0,70$ e o vetor de Fiedler de G é $(-0.20491, 0, -0.67677, 0.20491, 0.67677)$.

A Proposição I.24 relaciona o espectro da matriz laplaciana de G com o espectro do seu complementar $\overline{G}$.

Proposição I.24. [30] Se o espectro da matriz laplaciana de um grafo G é

$$Spec(L(G)) = ({\mu_1}^{m(\mu_1)} \quad \dots \quad {\mu_{n-1}}^{m(\mu_{n-1})} \quad 0),$$

então o espectro da matriz laplaciana de $\overline{G}$ é

$$Spec(L(\overline{G})) = (n - {\mu_{n-1}}^{m(\mu_{n-1})} \quad \dots \quad n - {\mu_1}^{m(\mu_1)} \quad 0).$$

As próximas proposições exibem o espectro da matriz laplaciana de alguns grafos que serão utilizados nos capítulos posteriores. Destacamos a Proposição I.28, [34], que tem fundamental importância para a prova do principal resultado teórico deste trabalho, descrito no Capítulo IV.

Proposição I.25. O espectro da matriz laplaciana do grafo completo K_n é

$$Spec(L(K_n)) = (n^{n-1} \quad 0).$$

Proposição I.26. O espectro da matriz laplaciana do grafo bipartido completo $K_{s,t}$, tal que $s \leq t$, é

$$Spec(L(K_{s,t})) = (s+t \quad t^{s-1} \quad s^{t-1} \quad 0).$$

Proposição I.27. Seja O_n um grafo totalmente desconexo, ou seja, sem arestas, com n vértices. Então, o espectro da matriz laplaciana de O_n é

$$Spec(L(O_n)) = (0^n).$$

Proposição I.28. Sejam G_1 e G_2 grafos com k e m vértices, respectivamente, com autovalores da matriz laplaciana $\mu_1(G_1) \geq \ldots \geq \mu_{k-1}(G_1) \geq \mu_k(G_1) = 0$ e $\mu_1(G_2) \geq \ldots \geq \mu_{m-1}(G_2) \geq \mu_m(G_2) = 0$. Então, os autovalores da matriz laplaciana de $G_1 \vee G_2$ são

$$m + k, \mu_1(G_1) + m, \ldots, \mu_{k-1}(G_1) + m, \mu_1(G_2) + k, \ldots, \mu_{m-1}(G_2) + k, 0.$$

Capítulo II - Remoção de um vértice em famílias de grafos e a conectividade algébrica

Iniciamos este capítulo apresentando a função $\phi(v)$, na Seção II.1, introduzida e estudada por Kirkland em [33]. Na Seção II.2, são apresentados resultados obtidos para esta função em algumas famílias de grafos, onde uma análise da função $\phi(v)$ é realizada.

II.1 Função de perturbação: resultados gerais

Sejam $G(V, E)$ um grafo conexo e $v \in V(G)$. $G \setminus v$ denota o subgrafo de G obtido eliminando-se o vértice v e todas as arestas incidentes nele. A função $\phi(v)$ é definida como sendo a diferença entre a conectividade algébrica de G, $a(G)$, e a conectividade algébrica de $G \setminus v$, $a(G \setminus v)$, ou seja,

$$\phi(v) = a(G) - a(G \setminus v).$$

Fiedler em 1973 provou que $a(G \setminus v) \geq a(G) - 1$. Neste caso, naturalmente obtém-se o resultado do Teorema II.1. A seguir são apresentados resultados envolvendo limites superiores e inferiores para $\phi(v)$ e uma caracterização quando $\phi(v) = 1$, que se encontram em [33].

Teorema II.1. [22] Seja $G(V, E)$ um grafo $\forall$ vértice $v \in V$. Então

$$\phi(v) \leq 1.$$

Antes de apresentarmos o próximo resultado, denotamos por $N(v)$ o conjunto dos vértices de G que são adjacentes a v e definimos o conjunto de índices correspondentes às entradas não-nulas de um vetor w como sendo o suporte de w.

Teorema II.2. [33] Seja $G(V, E)$ um grafo com $n \geq 3$ vértices, e seja $v \in V$. $\phi(v) = 1$ se e somente se existe um vetor de Fiedler de G cujo suporte é um subconjunto de $N(v)$.
Demonstração:

Primeiramente, suponhamos que $v \in V$ tal que $d(v) = m$ e $\phi(v) = a(G) - a(G \setminus v) = 1$. A matriz laplaciana de G pode ser escrita na forma

$$L(G) = \left[\begin{array}{c|c|c} L_{11} & L_{12} & 0 \\ \hline L_{21} & L_{22} + I & \textbf{-1} \\ \hline 0^T & \textbf{-1}^T & m \end{array}\right],$$

onde a última linha e coluna correspondem ao vértice v, e

$$L(G \setminus v) = \left[\begin{array}{c|c} L_{11} & L_{12} \\ \hline L_{21} & L_{22} \end{array}\right].$$

Seja w um vetor de Fiedler para $L(G \setminus v)$, tal que $w^T \cdot w = 1$, particionado da seguinte

maneira

$$\left[\begin{array}{c} w_1 \\ \hline w_2 \end{array}\right].$$

Tomemos um vetor $z \in \mathbb{R}^n$ da seguinte maneira

$$\left[\begin{array}{c} w_1 \\ \hline w_2 \\ \hline 0 \end{array}\right].$$

Logo,
$a(G \setminus v) \cdot z^T \cdot z + {w_1}^T \cdot w_1 + {w_2}^T \cdot w_2 = a(G \setminus v) \cdot z^T \cdot z + z^T \cdot z = (a(G \setminus v) + 1) \cdot z^T \cdot z = a(G) \cdot z^T \cdot z \leq z^T \cdot L(G) \cdot z = w^T \cdot L(G \setminus v) \cdot w + {w_2}^T \cdot w_2 = a(G \setminus v) \cdot w^T \cdot w + {w_2}^T \cdot w_2 = a(G \setminus v) \cdot z^T \cdot z + {w_2}^T \cdot w_2 \iff a(G \setminus v) \cdot z^T \cdot z + {w_1}^T \cdot w_1 + {w_2}^T \cdot w_2 \leq a(G \setminus v) \cdot z^T \cdot z + {w_2}^T \cdot w_2 \iff {w_1}^T \cdot w_1 \leq 0.$

Portanto, $w_1 = 0$ e z é um vetor de Fiedler de G, cujo suporte é um subconjunto de $N(v)$.

Agora, suponhamos que G tem um vetor de Fiedler y, $y^T \cdot y = 1$, cujo suporte é um subconjunto de $N(v)$. Podemos reescrever $L(G)$ da seguinte maneira

$$L(G) = \left[\begin{array}{c|c} \tilde{L} + D & -X \\ \hline -X^T & m \end{array}\right],$$

onde $\tilde{L}$ é a matriz laplaciana de $G \setminus v$ e D é uma matriz diagonal cujas entradas de 1´s correspondem aos vértices em $N(v)$, e zeros caso contrário. Assim, podemos escrever y da seguinte maneira

$$y = \left[\begin{array}{c} \tilde{y} \\ \hline 0 \end{array}\right].$$

Como $y^T \cdot L(G) \cdot y = a(G) \cdot y^T \cdot y$, temos que

$$\tilde{y}^T \cdot \tilde{L} \cdot \tilde{y} + \tilde{y}^T \cdot D \cdot \tilde{y} = a(G) \cdot \tilde{y}^T \cdot \tilde{y} \iff \tilde{y}^T \cdot \tilde{L} \cdot \tilde{y} = a(G) \cdot \tilde{y}^T \cdot \tilde{y} - \tilde{y}^T \cdot D \cdot \tilde{y} =$$

$$a(G) \cdot \tilde{y}^T \cdot \tilde{y} - \tilde{y}^T \cdot \tilde{y} = (a(G) - 1) \cdot \tilde{y}^T \cdot \tilde{y}. \quad \text{(II.1)}$$

De II.1 e utilizando que $a(G \setminus v) \cdot \tilde{y}^T \cdot \tilde{y} \leq \tilde{y}^T \cdot \tilde{L}(G) \cdot \tilde{y}$, conclui-se que

$$a(G \setminus v) \cdot \tilde{y}^T \cdot \tilde{y} \leq (a(G) - 1) \cdot \tilde{y}^T \cdot \tilde{y} \iff a(G \setminus v) \leq a(G) - 1 \iff$$

$$1 \leq a(G) - a(G \setminus v).$$

Mas como pelo Teorema II.1, $a(G) - a(G \setminus v) \leq 1$, tem-se $a(G) - a(G \setminus v) = 1$, ou seja, $\phi(v) = 1$.■

Como consequência do Teorema II.2, se tomarmos um grafo H e um vértice isolado v e construirmos o grafo $G = H \vee v$, então $a(G) = a(H) + 1$. Assim, para um grafo G com n vértices que tem um vértice universal v, temos que $\phi(v) = 1$, ou seja, a igualdade do Teorema II.2 é satisfeita, conforme observado por Kirkland [33].

O Teorema II.3 determina um limite inferior para $\phi(v)$ em grafos conexos.

Teorema II.3. [33] Seja G um grafo conexo com $n \geq 3$ vértices. Então para qualquer vértice v de G, $\phi(v) \geq -(n-2)$. A igualdade acontece se e somente se $G = (K_{n-2} \cup K_1) \vee K_1$ e v é o vértice pendente do grafo.

Demonstração:

Primeiramente, suponhamos que $G \backslash v \neq K_{n-1}$. Da Proposição I.19, tem-se que $a(G \backslash v) \leq (n-3)$. Daí, temos que $-a(G \setminus v) \geq -(n-3)$ e, consequentemente, $\phi(v) = a(G) - a(G \setminus v) \geq -(n-3) > -(n-2)$. Agora, suponhamos que $G \setminus v = K_{n-1}$, onde v tem grau $d(v)$. Observe que $\overline{G \setminus v}$ é um grafo estrela $K_{1,n-1-d(v)}$ e através da Proposição I.25, obtém-se que: $a(G) = d(v)$. Logo, $\phi(v) = a(G) - a(G \setminus v) = d(v) - (n-1) \geq -(n-2)$. Além disso, $\phi(v) = -(n-2)$ se e somente se $d(v) - n + 1 = -n + 2$, ou seja, $d(v) = 1$. Portanto, $G \setminus v = K_{n-1}$ e $d(v) = 1$, ou seja, $G = (K_{n-2} \cup K_1)$.■

Através dos Teoremas II.1 e II.3, para qualquer grafo G conexo com $n \geq 3$ vértices, tem-se que

$$-(n-2) \leq \phi(v) \leq 1.$$

II.2 Função de perturbação: resultados em algumas famílias de grafos

Nesta seção, são apresentados resultados sobre a função $\phi(v)$ nas famílias de grafos *split* completo, abacaxi (*pineapple*), caminho completo (*path-complete*) e em grafos obtidos através das operações $1-$soma, $2-$soma, ponte de ligação entre grafos completos e ponte de ligação entre estrelas.

Proposição II.4. Seja $SC(n,q)$ um grafo *split* completo. Então:

i) $a(SC(n,q)) = q$, se $n - q > 1$;

ii) $a(SC(n,q)) = n$, se $n - q = 1$.

Demonstração:

Seja $SC(n,q)$ um grafo *split* completo. É fácil observar que o grafo complementar de $SC(n,q)$ é o grafo $K_{n-q} \cup O_q$, onde O_q é o grafo de q vértices sem arestas.

Através da Proposição I.25, o espectro de $L(K_{n-q})$ é:

$$Spec(L(K_{n-q})) = ((n-q)^{(n-q)-1} \quad 0).$$

Logo, o espectro de $L(K_{n-q} \cup O_q)$ é:

$$Spec(L(K_{n-q} \cup O_q)) = ((n-q)^{(n-q)-1} \quad 0^{q+1}).$$

Utilizando a Proposição I.24, tem-se que:

$$Spec(L(SC(n,q))) = ((n-0)^q \quad (n-(n-q))^{(n-q)-1} \quad 0)$$

$$Spec(L(SC(n,q))) = (n^q \quad q^{(n-q)-1} \quad 0).$$

Se $n-q>1$, tem-se que $a(SC(n,q))=q$ e se $n-q=1$, tem-se que $a(SC(n,q))=n$. ■

Corolário II.5. Seja $SC(n,q)$ um grafo *Split* Completo. Então:

i) $a(SC(n-1,q-1))=q-1$, se $n-q>1$;
$a(SC(n-1,q-1))=n-1$, se $n-q=1$;

ii) $a(SC(n-1,q))=q$, se $n-q>2$;
$a(SC(n-1,q))=n-1$, se $n-q=2$.

Demonstração:

Segue-se imediatamente da Proposição II.4.

Proposição II.6. Seja o grafo abacaxi $PA(n,q)$ com $n\geq 3$ e $q\geq 2$. Então, $a(PA(n,q))=1$.
Demonstração:

Seja $PA(n,q)$ um grafo abacaxi, com $n\geq 3$ e $q\geq 2$. $SC(n-1,n-q)\cup O_1$ é o grafo complementar do $PA(n,q)$, conforme Figura II.1.

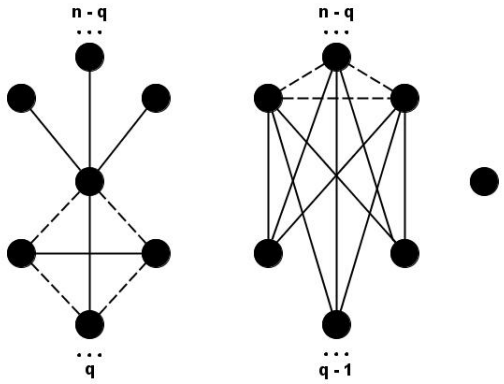


Figura II.1: $PA(n,q)$ e o seu complementar

De acordo com a Proposição II.4, tem-se que:

$$Spec(L(SC(n-1,n-q)))=((n-1)^{n-q}\ \ (n-q)^{(n-1)-(n-q)-1}\ \ 0).$$

Então,

$$Spec(L(SC(n-1,n-q)\cup O_1))=((n-1)^{n-q}\ \ (n-q)^{(n-1)-(n-q)-1}\ \ 0^2)=$$

$$=((n-1)^{n-q}\ \ (n-q)^{q-2}\ \ 0^2).$$

Utilizando a Proposição I.24 obtem-se que:

$$Spec(L(PA(n,q)))=(n-0\ \ (n-(n-q))^{q-2}\ \ (n-(n-1))^{n-q}\ \ 0)=$$

$$=(n\ \ q^{q-2}\ \ 1^{n-q}\ \ 0).$$

Logo, $a(PA(n,q)) = 1$. ■

Corolário II.7. Seja $PA(n,q)$ o grafo abacaxi, tal que $n \geq 4$, $q \geq 3$ e $n - q > 1$.

i) Se v pertence a clique, mas não é o vértice universal ou se v pertence ao conjunto independente, então $a(PA(n,q) \setminus v) = 1$;

ii) Se v é o vértice universal, então $a(PA(n,q) \setminus v) = 0$.

Demonstração:

As provas dos itens $i)$ e $ii)$ decorrem imediatamente das Proposições II.6 e I.27.■

Proposição II.8. Seja $PC_{n,p,1}$ o grafo caminho completo com $1 \leq p \leq n-2$. Então, $a(PC_{n,p,1}) = p$.
Demonstração:

O grafo complementar de $PC_{n,p,1}$ é $K_{1,n-p-1} \cup O_p$, conforme Figura II.2.

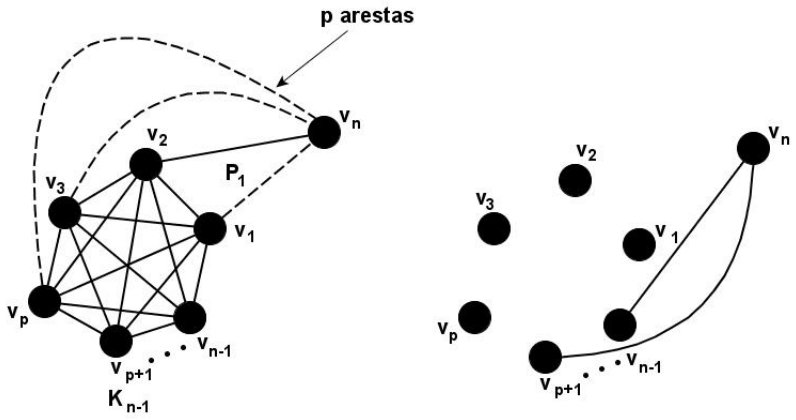


Figura II.2: $PC_{n,p,1}$ e seu complementar

O espectro de $L(K_{1,n-p-1})$, de acordo com a Proposição I.26, é

$$Spec(L(K_{1,n-p-1})) = (n-p \quad 1^{n-p-2} \quad 0).$$

Logo,

$$Spec(L(K_{1,n-p-1} \cup O_p)) = (n-p \quad 1^{n-p-2} \quad 0^{p+1}).$$

Utilizando a Proposição I.24, temos

$$Spec(L(PC_{n,p,1})) = ((n-0)^p \quad (n-1)^{n-p-2} \quad n-(n-p) \quad 0) = (n^p \quad (n-1)^{n-p-2} \quad p \quad 0).$$

Portanto, $a(PC_{n,p,1}) = p$.

Corolário II.9. Para $n \geq 4$, seja $PC_{n,p,1}$ o grafo caminho completo. Então:

i) Se v não pertence a clique, $a(PC_{n,p,1} \setminus v) = n-1$;

ii) Se v pertence a clique, mas não é o vértice universal e $1 \leq p < n-2$, $a(PC_{n,p,1} \setminus v) = p$;

iii) Se v pertence a clique, mas não é o vértice universal e $p = n-2$, $a(PC_{n,p,1} \setminus v) = n-1$;

iv) Se v pertence a clique e é o vértice universal, $a(PC_{n,p,1} \setminus v) = a(PC_{n-1,p-1,1}) = p-1$.

Demonstração:

Seja $PC_{n,p,1}$ o grafo caminho completo. Removendo o vértice não pertencente a clique, obtém-se o grafo K_{n-1} e, de acordo com a Proposição I.25, tem-se que

$$Spec(L(K_{n-1})) = ((n-1)^{n-1-1} \;\; 0) = ((n-1)^{n-2} \;\; 0).$$

Logo, $a(PC_{n,p,1} \setminus v) = n-1$.

Suponhamos, agora, que o vértice removido pertença a clique, mas não seja o universal e que $1 \leq p < n-2$. O grafo obtido é $PC_{n-1,p,1}$ e, de acordo com a Proposição II.8, tem-se que

$$Spec(L(PC_{n-1,p,1})) = ((n-1)^{p} \;\; (n-2)^{n-p-3} \;\; p \;\; 0).$$

Como $1 \leq p < n-2$, tem-se $a(PC_{n-1,p,1}) = a(PC_{n,p,1} \setminus v) = p$.

Suponhamos, ainda, que o vértice removido pertença a clique, mas não seja o universal e que $p = n-2$. O grafo obtido é K_{n-1} e, de acordo com a Proposição I.25 , tem-se que

$$a(K_{n-1}) = a(PC_{n-1,n-2,1} \setminus v) = (n-1).$$

Finalmente, suponhamos que v é o vértice universal. Logo, $a(PC_{n,p,1} \setminus v) = (PC_{n-1,p-1,1})$ e pela Proposição II.8, tem-se $a(PC_{n-1,p-1,1}) = p-1$. ■

A Proposição II.10 e o Corolário II.11 apresentam a conectividade algébrica dos grafos resultantes da operação $1-$soma entre grafos completos.

Proposição II.10. $a(K_r \circ K_p) = 1$, $\forall r \geq 2$ e $p \geq 2$.

Demonstração:

Seja $H = K_r \circ K_p$. Suponhamos que $r \leq p$. O grafo complementar do grafo H é o grafo $K_{r-1,p-1} \cup O_1$. Logo, pelas Proposições I.24 e I.26 tem-se:

$$Spec(L(K_{r-1,p-1} \cup O_1)) = ((r-1)+(p-1) \;\; (p-1)^{(r-1)-1} \;\; (r-1)^{(p-1)-1} \;\; 0^2)$$

$$= (r+p-2 \;\; (p-1)^{r-2} \;\; (r-1)^{p-2} \;\; 0^2).$$

e, consequentemente, $Spec(L(H))$ é dado por

$$((r+p-1)-0 \;\; [(r+p-1)-(r-1)]^{p-2} \;\; [(r+p-1)-(p-1)]^{r-2} \;\; (r+p-1)-(r+p-2) \;\; 0)$$

$$= (r+p-1 \;\; p^{p-2} \;\; r^{r-2} \;\; 1 \;\; 0).$$

Portanto, $a(H) = 1$. ■

Corolário II.11. Seja $K_r \circ K_p$, onde $3 \leq r \leq p$.

i) se v é o vértice de uma das cliques, mas não o universal, então $a(K_r \circ K_p \setminus v) = 1$;

ii) se v é o vértice universal, então $a(K_r \circ K_p \setminus v) = 0$.

Demonstração:

Suponhamos inicialmente que v pertence a clique K_r, mas não é o vértice universal. Daí temos $(K_r \circ K_p) \setminus v = K_{r-1} \circ K_p$.

Segue imediatamente pela Proposição II.10 que:

$$a(K_r \circ K_p \setminus v) = 1.$$

De maneira similar, obtemos a conectividade algébrica de $(K_r \circ K_p) \setminus v$ obtido pela remoção de um vértice v pertencente a clique K_p, mas que não seja o vértice universal e concluímos que $a(K_r \circ K_p \setminus v) = 1$.

Finalmente, suponhamos que v seja o vértice universal. Como $(K_r \circ K_p) \setminus v$ é desconexo, segue-se pela Proposição I.27 que $a(K_r \circ K_p \setminus v) = a(K_{r-1} \cup K_{p-1}) = 0$. ■

Os dois próximos reultados determinam a conectividade algébrica para os grafos resultantes da operação $2-$soma ou coalescência de arestas entre grafos completos.

Proposição II.12. $a(K_r \bullet K_p) = 2, \forall\ 3 \leq r \leq p$.
Demonstração:

Seja $H = K_r \bullet K_p$. É fácil observar que H tem $r+p-2$ vértices e que o grafo complementar do grafo H é o grafo $K_{r-2,p-2} \cup O_2$. Portanto, pelas Proposições I.24 e I.26, tem-se que:

$$Spec(L(K_{r-2,p-2} \cup O_2)) = ((r-2)+(p-2)\ \ (p-2)^{(r-2)-1}\ \ (r-2)^{(p-2)-1}\ \ 0^3) =$$

$$= ((r+p-4)\ \ (p-2)^{r-3}\ \ (r-2)^{p-3}\ \ 0^3).$$

E, consequentemente,

$$Spec(L(H)) = (((r+p-2)-0)^2\ \ ((r+p-2)-(r-2))^{p-3}\ \ ((r+p-2)-(p-2))^{r-3}$$

$$(r+p-2)-(r+p-4)\ \ 0) = ((r+p-2)^2\ \ p^{p-3}\ \ r^{r-3}\ \ 2\ \ 0).$$

Logo, $a(H) = 2$. ■

Corolário II.13. Seja $K_r \bullet K_p$, onde $4 \leq r \leq p$.

i) se v é o vértice de uma das cliques, mas não o universal, então $a(K_r \bullet K_p \setminus v) = 2$;

ii) se v é o vértice universal, então $a(K_r \bullet K_p \setminus v) = 1$.

Demonstração:

Seja v o vértice da clique de tamanho r, que não seja o vértice universal. Assim, temos $(K_r \bullet K_p) \setminus v = K_{r-1} \bullet K_p$. Segue imediatamente da Proposição II.12 que $a(K_r \bullet K_p \setminus v)$=2. De maneira similar, conclui-se que $a(K_r \bullet K_p \setminus v) = 2$ se v é o vértice da clique de tamanho p.

Se v é um dos vértices universais, o grafo resultante é uma coalescência de vértices, ou seja, $(K_r \bullet K_p) \setminus v = K_{r-1} \circ K_{p-1}$. Então, pela Proposição II.10 tem-se $a(K_r \bullet K_p \setminus v)$=1. ■

Como contribuição dessa dissertação, a Proposição II.14 e o Corolário II.15 apresentam a conectividade algébrica dos grafos resultantes da operação ponte de ligação entre grafos completos.

GUO em 2005, [19], provou que se $G = G_1u : vG_2$ é um grafo obtido através de um *join* entre o vértice u de G_1 ao vértice v de G_2 por uma aresta, então

$$p_{L(G)}(\lambda) = p_{L(G_1)}(\lambda) \cdot p_{L(G_2)}(\lambda) - p_{L(G_1)}(\lambda) \cdot p_{L_v(G_2)}(\lambda) - p_{L(G_2)}(\lambda) \cdot p_{L_u(G_1)}(\lambda). \tag{II.2}$$

Com base nesse resultado, a Proposição II.14 traz na equação II.3 o polinômio característico para os grafos resultantes da operação ponte de ligação entre grafos completos.

Proposição II.14. Seja $G = K_ru : K_pv$, onde $7 \leq r \leq p$. Então, $a(G) \in (0, 0.5)$. **Demonstração:**

Seja $G = K_ru : K_pv$, onde $r \leq p$. De acordo com [19], temos que

$$p_{L(G)}(\lambda) = p_{L(K_r)}(\lambda) \cdot p_{L(K_p)}(\lambda) - p_{L(K_r)}(\lambda) \cdot p_{L_v(K_p)}(\lambda) - p_{L(K_p)}(\lambda) \cdot p_{L_u(K_r)}(\lambda), \tag{II.3}$$

onde $L_u(K_r)$ e $L_v(K_p)$ são as matrizes obtidas através das matrizes laplacianas de K_r e K_p, eliminando-se a linha e a coluna referente ao vértice u e v, respectivamente. É fácil observar que

$$L(K_r) = [rI_r - J_r],$$

onde J_r é uma matriz de ordem r, cujas entradas são todas iguais a 1 e I_r a matriz identidade de ordem r. Rotulando o grafo de modo que u esteja na última linha de $L(K_r)$ obtemos

$$L_u(K_r) = [rI_{r-1} - J_{r-1}].$$

Além disso, podemos observar que

$$L_u(K_r) \cdot \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = 1 \cdot \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.$$

Logo, 1 é um autovalor de $L_u(K_r)$ com multiplicidade algébrica no mínimo 1. Tomemos os vetores $e_1 - e_i, \quad 2 \leq i \leq r-1$, onde e_i é o *i*-ésimo vetor de base canônica. Assim,

$$L_u(K_r) \cdot [e_1 - e_i] = r \cdot [e_1 - e_i].$$

Portanto, r é autovalor de $L_u(K_r)$ com multiplicidade algébrica no mínimo $r-2$. Logo,

$$p_{L_u(K_r)}(\lambda) = (\lambda - r)^{r-2} \cdot (\lambda - 1). \tag{II.4}$$

Utilizando o mesmo raciocínio para $L_v(K_p)$, conclui-se que

$$p_{L_v(K_p)}(\lambda) = (\lambda - p)^{p-2} \cdot (\lambda - 1). \tag{II.5}$$

Através da Proposição I.25 e de (II.4) e (II.5) podemos reescrever a equação (II.3) da seguinte maneira:

$p_{L(G)}(\lambda) = (\lambda - r)^{r-1} \cdot \lambda \cdot (\lambda - p)^{p-1} \cdot \lambda - (\lambda - r)^{r-1} \cdot \lambda \cdot (\lambda - p)^{p-2} \cdot (\lambda - 1) - (\lambda - p)^{p-1} \cdot \lambda \cdot (\lambda - r)^{r-2} \cdot (\lambda - 1)$

$p_{L(G)}(\lambda) = (\lambda - r)^{r-2} \cdot \lambda \cdot (\lambda - p)^{p-2} \cdot [(\lambda - r) \cdot \lambda \cdot (\lambda - p) - (\lambda - r) \cdot (\lambda - 1) - (\lambda - p) \cdot (\lambda - 1)]$

$p_{L(G)}(\lambda) = (\lambda - r)^{r-2} \cdot \lambda \cdot (\lambda - p)^{p-2} \cdot [(\lambda^2 - r\lambda) \cdot (\lambda - p) - (\lambda^2 - \lambda \cdot (r+1) + r) - (\lambda^2 - \lambda \cdot (p+1) + p)]$

$p_{L(G)}(\lambda) = (\lambda - r)^{r-2} \cdot \lambda \cdot (\lambda - p)^{p-2} \cdot [\lambda^3 + (-p - r - 2) \cdot \lambda^2 + (r + 1 + p + 1 + rp) \cdot \lambda - (r + p)]$

$p_{L(G)}(\lambda) = (\lambda - r)^{r-2} \cdot \lambda \cdot (\lambda - p)^{p-2} \cdot [\lambda^3 + (-p - r - 2) \cdot \lambda^2 + (r + p + 2 + rp) \cdot \lambda - (r + p)]$.

Consideremos o polinômio de grau 3 de $p_{L(G)}(\lambda)$ acima, dado por

$$p(\lambda) = \lambda^3 + (-p - r - 2) \cdot \lambda^2 + (r + p + 2 + rp) \cdot \lambda - (r + p).$$

Avaliando $p(\lambda)$ nos pontos 0 e $0,5$ temos que

i) $p(0) = -(r + p) < 0$;

ii) $p(0,5) = 0,5rp - 0,75(p + r) + 0,625$.

Como por hipótese $7 \leq r \leq p$, temos que

$$\frac{r}{\frac{0,5}{0,75}r - 1} < 2 < p \Leftrightarrow \frac{r}{\frac{0,5}{0,75}r - 1} < p \Leftrightarrow r < \frac{0,5}{0,75}rp - p \Leftrightarrow r + p < \frac{0,5}{0,75}rp \Leftrightarrow$$

$0,75(r + p) < 0,5rp \Rightarrow 0,5rp - 0,75(r + p) > 0 \Rightarrow 0,5rp - 0,75(r + p) + 0,625 > 0$.

Portanto, $p(0,5) > 0$. Como p é contínua em $[0, 0.5]$, temos que $a(K_r u : K_p v) \in (0, 0.5)$. ■

Corolário II.15. Seja $G = (K_r u : K_p v)$, onde $7 \leq r \leq p$.

i) se $w = u$ ou $w = v$, então $a(G \setminus w) = 0$;

ii) se $w \neq u$ e $w \neq v$, então $a(G \setminus w) \in (0, 0.5)$.

Demonstração:

Segue-se imediatamente das Proposições I.23 e II.14.■

A Proposição II.16 e o Corolário II.17 apresentam a conectividade algébrica dos grafos dupla estrela. Observe que esses grafos são um caso particular dos grafos pontes de ligação, pois $D_{p,q} = K_{1,p} u : K_{1,q} v$.

Proposição II.16. Seja $D_{p,q}$, onde $p \geq 2$ e $q \geq 3$. Então, $a(D_{p,q}) \in (0, 0.5)$.

Demonstração:

Como $D_{p,q} = K_{1,p}u : K_{1,q}v$, onde $p \geq 2$ e $q \geq 3$, de acordo com [19], temos que

$$p_{L(D_{p,q})}(\lambda) = p_{L(K_{1,p})}(\lambda) \cdot p_{L(K_{1,q})}(\lambda) - p_{L(K_{1,p})}(\lambda) \cdot p_{L_v(K_{1,q})}(\lambda) - p_{L(K_{1,q})}(\lambda) \cdot p_{L_u(K_{1,p})}(\lambda), \quad \text{(II.6)}$$

onde $L_v(K_{1,q})$ e $L_u(K_{1,p})$ são matrizes obtidas através das matrizes laplacianas de $K_{1,q}$ e $K_{1,p}$, eliminando a linha e a coluna referente ao vértice v e u, respectivamente. Como

$$L(K_{1,q}) = \begin{bmatrix} q & -1 & -1 & \dots & -1 \\ -1 & 1 & 0 & \dots & 0 \\ -1 & 0 & 1 & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ -1 & 0 & \dots & 0 & 1 \end{bmatrix},$$

é uma matriz de ordem $q+1$, temos que

$$L_v(K_{1,q}) = I_q.$$

Portanto, 1 é autovalor de $L_v(K_{1,q})$ com multiplicidade algébrica q. Desta forma,

$$p_{L_v(K_{1,q})}(\lambda) = (\lambda - 1)^q. \quad \text{(II.7)}$$

Utilizando o mesmo raciocínio para $L_u(K_{1,p})$, conclui-se que

$$p_{L_u(K_{1,p})}(\lambda) = (\lambda - 1)^p. \quad \text{(II.8)}$$

Através da Proposição I.26 e de (II.7) e (II.8) conclui-se que (II.6) pode ser escrito da seguinte maneira:
$p_{L(D_{p,q})}(\lambda) = \lambda \cdot (\lambda-1)^{p-1} \cdot (\lambda-(p+1)) \cdot \lambda \cdot (\lambda-1)^{q-1} \cdot (\lambda-(q+1)) - \lambda \cdot (\lambda-1)^{p-1} \cdot (\lambda-(p+1)) \cdot (\lambda-1)^q - \lambda \cdot (\lambda-1)^{q-1} \cdot (\lambda-(q+1)) \cdot (\lambda-1)^p \Leftrightarrow$
$p_{L(D_{p,q})}(\lambda) = \lambda \cdot (\lambda-1)^{p+q-2} \cdot [\lambda \cdot (\lambda^2 - \lambda \cdot (q+1) - \lambda \cdot (p+1) + (p+1) \cdot (q+1)) - (\lambda-(p+1)) \cdot (\lambda-1) - (\lambda-(q+1)) \cdot (\lambda-1)] \Leftrightarrow$
$p_{L(D_{p,q})}(\lambda) = \lambda \cdot (\lambda-1)^{p+q-2} \cdot [\lambda^3 + (-q-p-4) \cdot \lambda^2 + (pq+2p+2q+5) \cdot \lambda - (p+q+2)].$

Consideremos o polinômio de grau 3 na equação $p_{L(D_{p,q})}(\lambda)$ acima, dado por

$$p(\lambda) = \lambda^3 + (-q-p-4) \cdot \lambda^2 + (pq+2p+2q+5) \cdot \lambda - (p+q+2).$$

Avaliando $p(\lambda)$ nos pontos 0 e $0,5$ temos que

i) $p(0) = -(p+q+2) < 0$;

ii) $p(0,5) = 0,5pq - 0,25(p+q) - 0,375$.

Como por hipótese $p \geq 2$ e $q \geq 3$, temos que

$$\frac{p+1,5}{\frac{0,5}{0,25}p - 1} < 2 < q \Leftrightarrow \frac{p+1,5}{\frac{0,5}{0,25}p - 1} < q \Leftrightarrow p + 1,5 < \frac{0,5}{0,25}pq - q \Leftrightarrow p + q + 1,5 < \frac{0,5}{0,25}pq \Leftrightarrow$$

$0,25(p+q)+0,375 < 0,5pq \Rightarrow 0,5pq - 0,25(p+q) - 0,375 > 0.$

Portanto, $p(0,5) > 0$. Como p é contínua em $[0, 0.5]$, temos que $a(D_{p,q}) \in (0, 0.5)$■.

Corolário II.17. Seja $D_{p,q}$, onde $p \geq 2$ e $q \geq 3$.

i) se v é um vértice pendente, então $a(D_{p,q} \setminus v) \in (0, 0.5)$;

ii) se v não é um vértice pendente, então $a(D_{p,q} \setminus v) = 0$.

Demonstração:

Segue imediatamente da Proposição II.16 e Proposição I.23.

A Tabela II.1 resume os resultados obtidos para a função $\phi(v)$ nas famílias de grafos apresentadas nesta seção.

Grafo	$a(G)$	$a(G \setminus v)$	$\phi(v)$
$SC(n,q)$	q, se $n-q>1$	$1°caso: v \in K_q$ $q-1$, se $n-q>1$	 1
		$2°caso: v \notin K_q$ q, se $n-q>2$ $n-1$, se $n-q=2$	 0 -1
	n, se $n-q=1$	$v \in K_q$ $n-1$, se $n-q=1$	 1
$PA(n,q)$	1, se $n \geq 3$ e $q \geq 2$	0, se v é universal 1, caso contrário	1 0
$PC_{n,p,1}$	p	$1°caso: v \notin K_{n-1}$ $n-1$	 $p-n+1$
		$2°caso: v \in K_{n-1}$ $p-1$, se v é universal caso contrário, p, se $1 \leq p < n-2$ $n-1$, se $p=n-2$	 1 0 -1
$K_r \circ K_p$	1, se $r \geq 2$ e $p \geq 2$	0, se v é universal 1, caso contrário	1 0
$K_r \bullet K_p$	2, se $p \geq r \geq 3$	1, se v é universal 2, caso contrário	1 0
$(K_r u : K_p v)$	$(0, 0.5)$, se $p \geq r \geq 7$	0, se $w=u$ ou $w=v$ $(0, 0.5)$, se $w \neq u$ e $w \neq v$	$\in (0, 0.5)$ $\in [0, 0.5)$
$D_{p,q}$	$(0, 0.5)$, se $p \geq 1$ e $q \geq 3$	$(0, 0.5)$, se v é pendente 0, caso contrário	$\in [0, 0.5)$ $\in (0, 0.5)$

Tabela II.1: Resultados obtidos para $\phi(v)$

De acordo com a Tabela II.1, verifica-se que $\phi(v)$ é igual a zero ou a 1 dentro de algumas famílias estudadas. Kirkland em [33] caracterizou todos os casos em que $\phi(v) = 1$, porém o caso em que $\phi(v) = 0$ ainda não foi caracterizado, tornando-se um dos objetos de pesquisa dessa tese. Os resultados obtidos encontram-se nos Capítulos III e IV.

Capítulo III - Função de perturbação em grafos que possuem pelo menos um vértice de grau $(n-2)$

Kirkland em [33] mostrou que, se G tem um vértice universal v e este vértice é removido, tem-se $\phi(v) = 1$. A partir desse fato, o autor levantou a seguinte questão: o que acontece com a função $\phi(v)$ se o grafo G possui pelo menos um vértice v de grau $(n-2)$ e este é removido? Com base nessa questão, primeiramente selecionamos dentro das famílias estudadas no Capítulo II e dentro de todos os grafos conexos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$, grafos que possuem pelo menos um vértice de grau $(n-2)$. A partir daí, realizamos uma análise da função $\phi(v)$ quando o vértice de grau $(n-2)$ é removido, a qual é apresentada nas Seções III.1 e III.2.

III.1 *Split* Completo, Caminho Completo, Dupla estrela

Nesta seção, apresentamos a análise da função $\phi(v)$ nos grafos que possuem pelo menos um vértice de grau $(n-2)$ dentre as famílias apresentadas no Capítulo II.

Os grafos *split* completo, que possuem pelo menos um vértice de grau $(n-2)$, são os grafos $SC(n, n-2) = K_n - \{e\}$, onde e é uma aresta. É fácil observar que os grafos $SC(n, n-2)$ possuem:

- 2 vértices de grau $(n-2)$;
- $(n-2)$ vértices de grau $(n-1)$.

Assim, através da Proposição II.4 temos $a(SC(n, n-2)) = n-2$ e, além disso, $a(SC(n, n-2 \backslash v)) = a(K_{n-1}) = n-1$, onde v é um dos vértices de grau $(n-2)$. Logo,

$$\phi(v) = a(SC(n, n-2)) - a(SC(n, n-2 \setminus v)) = -1.$$

Já os grafos caminho completo, que possuem pelo menos um vértice de grau $(n-2)$, são os grafos $PC_{n,p,1}$, com $1 \leq p \leq n-2$. Portanto, esses grafos possuem:

- 1 vértice de grau p;
- p vértice(s) de grau $(n-1)$;
- $n-1-p$ vértice(s) de grau $(n-2)$.

De acordo com a Proposição II.8, temos que $a(PC_{n,p,1}) = p$, e do Corolário II.9, há dois casos de remoção do vértice de grau $(n-2)$ a serem considerados, os quais são apresentados abaixo.
Caso 1: para $1 \leq p < n-2$, temos $a(PC_{n,p,1} \setminus v) = p$. Logo,

$$\phi(v) = a(PC_{n,p,1}) - a(PC_{n,p,1} \setminus v) = 0.$$

Caso 2: para $p = n-2$, temos $a(PC_{n,p,1} \setminus v) = n-1$. Logo,

$$\phi(v) = a(PC_{n,p,1}) - a(PC_{n,p,1} \setminus v) = p - n + 1 = -1.$$

Agora, analisemos os grafos dupla estrela. Nesta família, os grafos $D_{1,q}$, com $q \geq 3$, que possuem vértices de grau $(n-2)$, são apresentados na Figura III.1.

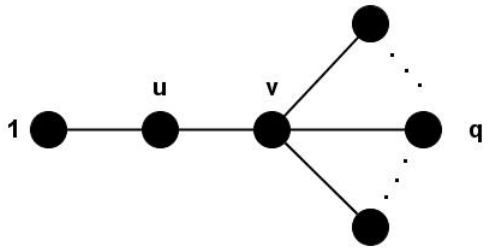


Figura III.1: $D_{1,q}$, com $q \geq 3$

De acordo com a Figura III.1, os $D_{1,q}$ possuem:

- $q+1$ vértices de grau 1;
- 1 vértice de grau 2;
- 1 vértice de grau $q+1$.

Assim, através da Proposição II.16 e do Corolário II.17, temos que $a(D_{1,q}) \in (0, 0.5)$ e $a(D_{1,q} \backslash v) = 0$. Logo,

$$\phi(v) = a(D_{1,q}) - a(D_{1,q} \setminus v) \in (0, 0.5).$$

As outras famílias de grafos estudadas no Capítulo II, que possuem pelo menos um vértice de grau $(n-2)$, são casos particulares do grafo caminho completo, $PC_{n,p,1}$.

A Tabela III.1 resume os resultados obtidos para a função $\phi(v)$ nesta seção.

Grafo	$a(G)$	$a(G \setminus v)$	$\phi(v)$
$SC(n, n-2)$	$n-2$	$n-1$	-1
$PC_{n,p,1}$	p	p, com $1 \leq p < n-2$	0
		$n-1$, com $p = n-2$	-1
$D_{1,q}$	$(0, 0.5)$, com $q \geq 3$	0	$\in (0, 0.5)$

Tabela III.1: Resultados para $\phi(v)$

III.2 Grafos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$

Nesta seção, apresentamos o estudo realizado para a função $\phi(v)$ em grafos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$, que possuem pelo menos um vértice de grau $(n-2)$. Todos esses grafos e seus subgrafos são desenhados no Apêndice A. Para esses grafos, apresentamos o espectro, o espectro do subgrafo obtido a partir da remoção do vértice v de grau $(n-2)$ e o valor da função $\phi(v)$. Os resultados foram obtidos com o auxílio do AGX [16] e podem ser encontrados no Apêndice B. Com base nesses dados, selecionamos para a próxima etapa de nossa pesquisa os 46 grafos G para os quais existe um vértice v de grau $(n-2)$ tal que $\phi(v) = 0$,

com o objetivo de analisarmos a estrutura desses grafos e observarmos o comportamento do vetor de Fiedler e a conectividade algébrica.

Capítulo IV - Grafos com função de perturbação nula que possuem pelo menos um vértice de grau $(n-2)$

O objetivo deste capítulo é explorar a questão: "O que pode-se dizer sobre $\phi(v)$ quando v tem grau $(n-2)$?", levantada por Kirkland em 2010 no trabalho [33]. Este problema, em aberto na literatura, foi resolvido neste trabalho quando $\phi(v) = 0$. Observamos que os grafos para os quais $\phi(v) = 0$ onde $d(v) = n-2$ apresentam necessariamente a estrutura de um *join* de 4 grafos. Discutiremos aqui a estrutura de tais grafos e a prova é apresentada na Proposição IV.3 da Seção IV.3. A estrutura de tais grafos e o comportamento do vetor de Fiedler, inicialmente, foram observados nos testes computacionais. A partir dessa observação, verificamos tal comportamento na família dos grafos caminho completo a qual é apresentado no início da próxima seção.

IV.1 Grafos caminho completo

Os grafos caminho completo, conforme Definição I.4 possuem pelo menos um vértice de grau $(n-2)$. Em particular todos os grafos do subconjunto $PC_{n,p,1}$, que é o conjunto dos grafos caminho completo com n vértices e um vértice do caminho ligado a clique K_{n-1} por p arestas, possuem $n-p-1$ vértices de grau $(n-2)$, onde $1 \leq p \leq n-2$. Já provamos na Proposição II.8 que se $G = PC_{n,p,1}$, então $a(G) = p$. Além disso, se um vértice v da clique K_{n-1} com grau $(n-2)$ é removido, tem-se $a(G \setminus v) = p$ e, portanto, $\phi(v) = 0$. O resultado a seguir, mostra que o autovetor de Fiedler desta classe de grafos segue um certo padrão.

Proposição IV.1. Sejam $G = PC_{n,p,1}$ e $v = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ -1 \\ \hline n-p-1 \\ \vdots \\ -1 \\ \hline n-p-1 \end{bmatrix}$. Então, v é um vetor de Fiedler.

Demonstração:

Seja $v = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ -1 \\ \hline n-p-1 \\ \vdots \\ -1 \\ \hline n-p-1 \end{bmatrix}$. Com uma rotulação conveniente dos vértices de G,

pode-se escrever a matriz laplaciana como

$$L(G) = \begin{bmatrix} p & -1 & -1 & \dots & -1 & 0 & \dots & 0 \\ -1 & n-1 & -1 & \dots & -1 & -1 & \dots & -1 \\ \vdots & & \ddots & & & & & \\ \vdots & & & \ddots & & & & \\ -1 & -1 & \dots & \dots & n-1 & -1 & \dots & -1 \\ 0 & -1 & \dots & \dots & -1 & n-2 & -1 & -1 \\ \vdots & & & & & & \ddots & \\ 0 & -1 & \dots & -1 & -1 & \dots & -1 & n-2 \end{bmatrix}.$$

Daí, temos

$$L(G)\cdot v = \begin{bmatrix} p & -1 & -1 & \dots & -1 & 0 & \dots & 0 \\ -1 & n-1 & -1 & \dots & -1 & -1 & \dots & -1 \\ \vdots & & \ddots & & & & & \\ \vdots & & & \ddots & & & & \\ -1 & -1 & \dots & \dots & n-1 & -1 & \dots & -1 \\ 0 & -1 & \dots & \dots & -1 & n-2 & -1 & -1 \\ \vdots & & & & & & \ddots & \\ 0 & -1 & \dots & -1 & -1 & \dots & -1 & n-2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \frac{-1}{n-p-1} \\ \vdots \\ \frac{-1}{n-p-1} \end{bmatrix} =$$

$$\begin{bmatrix} p \\ 0 \\ \vdots \\ 0 \\ \frac{-p}{n-p-1} \\ \vdots \\ \frac{-p}{n-p-1} \end{bmatrix} = p \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \frac{-1}{n-p-1} \\ \vdots \\ \frac{-1}{n-p-1} \end{bmatrix} = a(G)\cdot v.$$

Portanto, v é um vetor de Fiedler.■

IV.2 Grafos gerais

Nesta seção, observamos o comportamento do vetor de Fiedler e a conectividade algébrica para todos os grafos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$. Isto nos levou a uma análise computacional de um total de 505 grafos. A ferramenta OCTAVE, [20], que é um programa livre e similar ao Matlab, foi utilizado no cálculo dos autovalores e autovetores dos grafos. As tabelas a seguir exibem os 46 grafos com função de perturbação $\phi(v) = 0$ de forma que $d(v) = n - 2$. Os resultados aqui obtidos foram fundamentais para descrevermos os grafos com $\phi(v) = 0$, onde $d(v) = n - 2$ e para a obtenção dos principais resultados teóricos apresentados

na próxima seção.

Tabela IV.1: Grafos com 4 vértices

$Grafo$	$Spec$	$Vetor\ de\ Fiedler$
G_2	$(4\ 3\ 1\ 0)$	$(0.81650, 0, -0.40825, -0.40825)$
$G_2 \setminus v_1$	$(3\ 1\ 0)$	$(-0.70711, 0, 0.70711)$

Tabela IV.2: Grafos com 5 vértices

$Grafo$	$Spec$	$Vetor\ de\ Fiedler$
G_9	$(5\ 4\ 2\ 1\ 0)$	$(-0.86603, 0, 0.28868, 0.28868, 0.28868)$
$G_9 \setminus v_1$	$(4\ 1^2\ 0)$	$(-0.40825, 0, -0.40825, 0.81650)$
		$(0.70711, 0, -0.70711, 0)$
G_{13}	$(5\ 4^2\ 1\ 0)$	$(-0.86603, 0, 0.28868, 0.28868, 0.28868)$
$G_{13} \setminus v_1$	$(4\ 3\ 1\ 0)$	$(0.81650, 0, -0.40825, -0.40825)$
G_{15}	$(5\ 4\ 3\ 2\ 0)$	$(0.81650, 0, 0, -0.40825, -0.40825)$
$G_{15} \setminus v_1$	$(4\ 2^2\ 0)$	$(-0.70711, 0, 0, 0.70711)$
		$(0, -0.70711, 0.70711, 0)$
G_{16}	$(5^2\ 4\ 2\ 0)$	$(0.81650, 0, 0, -0.40825, -0.40825)$
$G_{16} \setminus v_1$	$(4^2\ 2\ 0)$	$(0, -0.70711, 0.70711, 0)$

Tabela IV.3: Grafos com 6 vértices

$Grafo$	$Spec$	$Vetor\ de\ Fiedler$
G_{32}	$(6\ 5\ 2^2\ 1\ 0)$	$(0.89443, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
$G_{32} \setminus v_1$	$(5\ 1^3\ 0)$	$(-0.21132, 0, -0.57735, 0.78868, 0)$
		$(0.78868, 0, -0.57735, -0.21132, 0)$
		$(-0.28868, 0, -0.28868, -0.28868, 0)$
G_{45}	$(6\ 5\ 4\ 2\ 1\ 0)$	$(-0.89443, 0, 0.22361, 0.22361, 0.22361, 0.22361)$
$G_{45} \setminus v_1$	$(5\ 3\ 1^2\ 0)$	$(-0.28868, 0, 0.86603, -0.28868, -0.28868)$
		$(0.81650, 0, -0.40825, -0.40825, 0)$
G_{59}	$(6\ 5^2\ 3\ 1\ 0)$	$(0.89443, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
$G_{59} \setminus v_1$	$(5\ 4\ 2\ 1\ 0)$	$(0.86603, 0, -0.28868, -0.28868, -0.28868)$
G_{67}	$(6\ 5\ 4\ 3\ 2\ 0)$	$(0.86603, 0, 0, -0.28868, -0.28868, -0.28868)$
$G_{67} \setminus v_2$	$(5\ 3\ 2^2\ 0)$	$(-0.40825, 0, 0, -0.40825, 0.81650)$

		$(0.70711, 0, 0, -0.70711, 0)$
G_{71}	$(6\ 5^3\ 1\ 0)$	$(0.89443, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
$G_{71} \setminus v_1$	$(5\ 4^2\ 1\ 0)$	$(-0.86603, 0, 0.28868, 0.28868, 0.28868)$
G_{72}	$(6^2\ 5\ 3\ 2\ 0)$	$(0.86603, 0, 0, -0.28868, -0.28868, -0.28868)$
$G_{72} \setminus v_1$	$(5^2\ 2^2\ 0)$	$(-0.15392, 0, 0, 0.77139, -0.61747)$
		$(-0.80186, 0, 0, 0.26763, 0.53423)$
G_{74}	$(6\ 5^2\ 4\ 2\ 0)$	$(-0.86603, 0, 0, 0.28868, 0.28868, 0.28868)$
$G_{74} \setminus v_1$	$(5\ 4\ 3\ 2\ 0)$	$(0.81650, 0, 0, -0.40825, -0.40825)$
G_{79}	$(6^2\ 5^2\ 2\ 0)$	$(0.86603, 0, 0, 0.28868, 0.28868, 0.28868)$
$G_{79} \setminus v_1$	$(5^2\ 4\ 2\ 0)$	$(0.81650, 0, 0, -0.40825, -0.40825)$
G_{81}	$(6^2\ 5\ 4\ 3\ 0)$	$(0.81650, 0, 0, 0, -0.40825, -0.40825)$
$G_{81} \setminus v_1$	$(5^2\ 3^2\ 0)$	$(-0.57533, 0.41108, 0, -0.41108, 0.57533)$
		$(0.41108, 0.57533, 0, -0.57533, -0.41108)$
G_{83}	$(6^3\ 5\ 3\ 0)$	$(0.81650, 0, 0, 0, -0.40825, -0.40825)$
$G_{83} \setminus v_1$	$(5^3\ 3\ 0)$	$(-0.70711, 0, 0, 0, 0.70711)$

Tabela IV.4: Grafos com 7 vértices

$Grafo$	$Spec$	$Vetor\ de\ Fiedler$
G_{169}	$(7\ 6\ 4\ 2^2\ 1\ 0)$	$(-0.91287, 0, 0.18257, 0.18257, 0.18257, 0.18257, 0.18257)$
$G_{169} \setminus v_1$	$(6\ 3\ 1^3\ 0)$	$(-0.84573, 0, 0.35993, 0.35993, -0.03121, 0.15708)$
		$(-0.07291, 0, -0.28784, -0.28784, -0.23187, 0.88046)$
		$(-0.28183, 0, -0.29597, -0.29597, 0.86329, 0.01048)$
G_{228}	$(7\ 6\ 5\ 3\ 2\ 1\ 0)$	$(-0.91287, 0, -0.18257, -0.18257, -0.18257, -0.18257, -0.18257)$
$G_{228} \setminus v_1$	$(6\ 4\ 2\ 1^2\ 0)$	$(0.75040, 0, 0.28366, -0.34469, -0.34469, -0.34469)$
		$(0.48672, 0, -0.84825, 0.12051, 0.12051, 0.12051)$
G_{237}	$(7\ 6\ 4^2\ 2\ 1\ 0)$	$(-0.91287, 0, 0.18257, 0.18257, 0.18257, 0.18257, 0.18257)$
$G_{237} \setminus v_1$	$(6\ 3^2\ 1^2\ 0)$	$(0.87899, 0, -0.12724, -0.12724, -0.31225, -0.31225)$
		$(0.16548, 0, -0.53274, -0.53274, 0.45000, 0.45000)$
G_{308}	$(7\ 6\ 5^2\ 2\ 1\ 0)$	$(-0.91287, 0, 0.18257, 0.18257, 0.18257, 0.18257, 0.18257)$
$G_{308} \setminus v_1$	$(6\ 4^2\ 1^2\ 0)$	$(0.67069, 0, 0.40529, -0.35866, -0.35866, 0.35866)$
		$(0.59175, 0, -0.79733, 0.68527, 0.68527, 0.68527)$
G_{309}	$(7\ 6^2\ 3^2\ 1\ 0)$	$(0.91287, 0, -0.18257, -0.18257, -0.18257, -0.18257, -0.18257)$
$G_{309} \setminus v_1$	$(6\ 5\ 2^2\ 1\ 0)$	$(0.89443, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
G_{310}	$(7\ 6\ 5.41\ 4\ 2.59\ 1\ 0)$	$(0.91287, 0, -0.18257, -0.18257, -0.18257, -0.18257, -0.18257)$
$G_{310} \setminus v_1$	$(6\ 4.41\ 3\ 1.59\ 1\ 0)$	$(0.89443, 0, -0.22361, -, 0.22361, -0.22361, -0.22361)$
G_{335}	$(7\ 6\ 5\ 3^2\ 2\ 0)$	$(-0.89443, 0, 0, 0.22361, 0.22361, 0.22361, 0.22361)$
$G_{335} \setminus v_3$	$(6\ 4\ 2^3\ 0)$	$(-0.44023, 0, 0, -0.44023, 0.10497, 0.77549)$
		$(0.23707, 0, 0, 0.23707, -0.85964, 0.38551)$

		$(0.70711, 0, 0, -0.70711, 0, 0)$
G_{380}	$(7\ 6^2\ 5\ 3\ 1\ 0)$	$(-0.91287, 0, 0.18257, 0.18257, 0.18257, 0.18257, 0.18257)$
$G_{380} \setminus v_1$	$(6\ 5\ 4\ 2\ 1\ 0)$	$(0.89443, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
G_{382}	$(7\ 6^2\ 4^2\ 1\ 0)$	$(-0.91287, 0, 0.18257, 0.18257, 0.18257, 0.18257, 0.18257)$
$G_{382} \setminus v_1$	$(6\ 5\ 3^2\ 1\ 0)$	$(-0.89443, 0, 0.22361, 0.22361, 0.22361, 0.22361)$
G_{387}	$(7^2\ 6\ 3^2\ 2\ 0)$	$(0.89443, 0, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
$G_{387} \setminus v_1$	$(6^2\ 2^3\ 0)$	$(0.25068, 0, 0, -0.00067, -0.79802, -0.54802)$
		$(0.81953, 0, 0, -0.17589, -0.10934, -0.53431)$
		$(-0.12463, 0, 0, 0.84798, -0.31813, -0.40521)$
G_{398}	$(7\ 6\ 5^2\ 3\ 2\ 0)$	$(-0.89443, 0, 0, 0.22361, 0.22361, 0.22361, 0.22361)$
$G_{398} \setminus v_3$	$(6\ 4^2\ 2^2\ 0)$	$(-0.29407, 0, 0, -0.66996, 0.48201, 0.48201)$
		$(0.81457, 0, 0, -0.54877, -0.13290, -0.13290)$
G_{431}	$(7\ 6\ 4^3\ 3\ 0)$	$(0.86603, 0, 0, 0, -0.28868, -0.28868, -0.28868)$
$G_{431} \setminus v_1$	$(6\ 3^4\ 0)$	$(-0.40825, 0, 0, 0, -0.40825, 0.81650)$
		$(0, 0.78868, -0.21132, -0.57735, 0, 0)$
		$(0.50000, 0.14943, -0.55768, 0.40825, -0.50000, 0)$
		$(-0.50000, 0.14943, -0.55768, 0.40825, 0.50000, 0)$
G_{432}	$(7\ 6^3\ 4\ 1\ 0)$	$(-0.91287, 0, 0.18257, 0.18257, 0.18257, 0.18257, 0.18257)$
$G_{432} \setminus v_1$	$(6\ 5^2\ 3\ 1\ 0)$	$(0.89443, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
G_{434}	$(7^2\ 6\ 5\ 3\ 2\ 0)$	$(0.89443, 0, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
$G_{434} \setminus v_1$	$(6^2\ 4\ 2^2\ 0)$	$(0.81573, 0, 0, -0.40901, -0.40901, 0.00229)$
		$(-0.29082, 0, 0, -0.28760, -0.28760, 0.86602)$
G_{441}	$(7\ 6^2\ 5\ 4\ 2\ 0)$	$(-0.89443, 0, 0, 0.22361, 0.22361, 0.22361, 0.22361)$
$G_{441} \setminus v_4$	$(6\ 5\ 4\ 3\ 2\ 0)$	$(-0.86603, 0, 0, 0.28868, 0.28868, 0.28868)$
G_{459}	$(7\ 6^2\ 4^2\ 3\ 0)$	$(0.86603, 0, 0, 0, -0.28868, -0.28868, -0.28868)$
$G_{459} \setminus v_1$	$(6\ 5\ 3^3\ 0)$	$(0.52238, -0.47419, 0.23709, 0.23709, 0.094761, 0.61714)$
		$(-0.55023, -0.60324, 0.30162, 0.30162, 0.27802, 0.27221)$
		$(0.30171, -0.27913, 0.13956, 0.13956, -0.76183, 0.46012)$
G_{462}	$(7\ 6^2\ 4^2\ 3\ 0)$	$(-0.86603, 0, 0, 0, 0.28868, 0.28868, 0.28868)$
$G_{462} \setminus v_1$	$(6\ 5\ 3^3\ 0)$	$(0.68518, -0.39282, 0.37577, 0.01706, -0.34259, -0.34259)$
		$(-0.44407, -0.60610, 0.57978, 0.02632, 0.22204, 0.22204)$
		$(0, -0.38079, -0.43511, 0.81589, 0, 0)$
G_{467}	$(7\ 6^4\ 1\ 0)$	$(-0.91287, 0, 0.18257, 0.18257, 0.18257, 0.18257, 0.18257)$
$G_{467} \setminus v_1$	$(6\ 5^3\ 1\ 0)$	$(0.89443, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
G_{468}	$(7^2\ 6^2\ 4\ 2\ 0)$	$(0.89443, 0, 0, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
$G_{468} \setminus v_1$	$(6^2\ 5\ 3\ 2\ 0)$	$(0.86603, 0, 0, -0.28868, -0.28868, -0.28868)$
G_{470}	$(7\ 6^3\ 5\ 2\ 0)$	$(-0.89443, 0, 0, 0.22361, 0.22361, 0.22361, 0.22361)$
$G_{470} \setminus v_6$	$(6\ 5^2\ 4\ 2\ 0)$	$(-0.86603, 0, 0, 0.28868, 0.28868, 0.28868)$
G_{478}	$(7^2\ 6\ 5\ 4\ 3\ 0)$	$(-0.86603, 0, 0, 0, 0.28868, 0.28868, 0.28868)$
$G_{478} \setminus v_3$	$(6^2\ 4\ 3^2\ 0)$	$(0.78665, 0, 0, 0, -0.20387, -0.58277)$
		$(-0.21876, 0, 0, 0, 0.79063, -0.57187)$

G_{480}	$(7\ 6^3\ 4\ 3\ 0)$	$(0.86603, 0, 0, 0, -0.28868, -0.28868, -0.28868)$
$G_{480} \setminus v_5$	$(6\ 5^2\ 3^2\ 0)$	$(0.81634, -0.00812, -0.00812, 0.01624, -0.40817, -0.40817)$
		$(-0.01624, -0.40817, -0.40817, 0.81634, 0.00812, 0.00812)$
G_{488}	$(7^2\ 6^3\ 2\ 0)$	$(0.89443, 0, 0, -0.22361, -0.22361, -0.22361, -0.22361)$
$G_{488} \setminus v_1$	$(6^2\ 5^2\ 2\ 0)$	$(-0.86603, 0, 0, 0.28868, 0.28868, 0.28868)$
G_{489}	$(7^3\ 6\ 4\ 3\ 0)$	$(-0.86603, 0, 0, 0, 0.28868, 0.28868, 0.28868)$
$G_{489} \setminus v_1$	$(6^3\ 3^2\ 0)$	$(0.54239, 0, 0, 0, 0.25735, -0.79974)$
		$(-0.61031, 0, 0, 0, 0.77488, -0.16457)$
G_{491}	$(7^2\ 6^2\ 5\ 3\ 0)$	$(0.86603, 0, 0, 0, -0.28868, -0.28868, -0.28868)$
$G_{491} \setminus v_5$	$(6^2\ 5\ 4\ 3\ 0)$	$(0.81650, 0, 0, 0, -0.40825, -0.40825)$
G_{496}	$(7^2\ 6\ 5^2\ 4\ 0)$	$(-0.81650, 0, 0, 0, 0, 0.40825, 0.40825)$
$G_{496} \setminus v_3$	$(6^2\ 4^3\ 0)$	$(-0.70711, 0, 0, 0, 0, 0.70711)$
		$(-0.67082, 0.22361, -0.22361, 0.67082, 0)$
		$(0, -0.22361, -0.67082, 0.67082, 0.22361, 0)$
G_{497}	$(7^3\ 6^2\ 3\ 0)$	$(-0.86603, 0, 0, 0, 0.28868, 0.28868, 0.28868)$
$G_{497} \setminus v_1$	$(6^3\ 5\ 3\ 0)$	$(0.81650, 0, 0, 0, -0.40825, -0.40825)$
G_{498}	$(7^4\ 6\ 4\ 0)$	$(0.81650, 0, 0, 0, 0, -0.40825, -0.40825)$
$G_{498} \setminus v_1$	$(6^4\ 4\ 0)$	$(-0.70711, 0, 0, 0, 0, 0.70711)$
G_{500}	$(7^3\ 6\ 5\ 4\ 0)$	$(-0.81650, 0, 0, 0, 0, 0.40825, 0.40825)$
$G_{500} \setminus v_4$	$(6^3\ 4^2\ 0)$	$(-0.61850, 0, 0, -0.34272, 0.34272, 0.61850)$
		$(-0.34272, 0, 0, 0.61850, -0.61850, 0.34272)$
G_{502}	$(7^4\ 6\ 4\ 0)$	$(0.81650, 0, 0, 0, 0, -0.40825, -0.40825)$
$G_{502} \setminus v_1$	$(6^4\ 4\ 0)$	$(-0.70711, 0, 0, 0, 0, 0.70711)$
G_{505}	$(7\ 6\ 2^3\ 1\ 0)$	$(0.91287, 0, -0.18257, -0.18257, -0.18257, -0.18257, -0.18257)$
$G_{505} \setminus v_1$	$(6\ 1^4\ 0)$	$(-0.16667, 0, -0.16667, -0.5, 0.83333, 0)$
		$(-0.16667, 0, 0.83333, -0.5, -0.16667, 0)$
		$(0.83333, 0, -0.16667, -0.5, -0.16667, 0)$
		$(0.22361, 0, 0.22361, 0.22361, 0.22361, -0.89443)$

A partir destas análises, alguns resultados foram obtidos e são apresentados na próxima seção.

IV.3 Resultados Teóricos

Com base nos experimentos computacionais, apresentamos, a seguir, as principais contribuições teóricas dessa dissertação. Antes, porém, definimos uma classe de grafos $\Im_{n,d}$.

Definição IV.2. Seja $\Im_{n,d}$ a classe de todos os grafos conexos com n vértices e com pelo menos um vértice v de grau d.

Neste trabalho, considere $d = n - 2$.

Tomemos $G \in \Im_{n,n-2}$ e v um vértice de grau $(n-2)$. Denote v_1 como o vértice de G, que não é adjacente a v, $(v_1 \nsim v)$. Assim, podemos particionar o conjunto de vértices V de G da seguinte

maneira $V_1 = \{v_1\}$, $V_2 = \{v_i \in V / v_i \sim v_1 \ e \ v_i \sim v\}$, $V_3 = \{v_i \in V / v_i \nsim v_1 \ e \ v_i \sim v\}$ e $V_4 = \{v\}$, onde

$$V = V_1 \cup V_2 \cup V_3 \cup V_4.$$

A Figura IV.1 ilustra a estrutura de um grafo $G \in \Im_{n,n-2}$ com as partições V_1 a V_4.

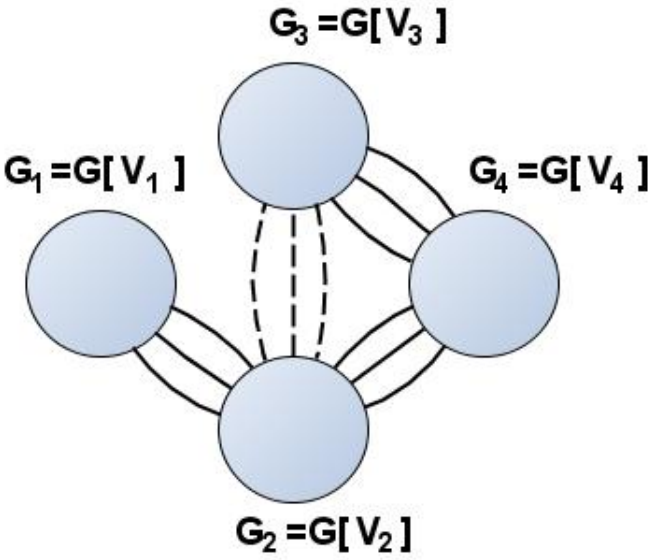


Figura IV.1: Estrutura de um grafo $G \in \Im_{n,n-2}$

Os grafos induzidos pelas partições $V_i, 1 \leq i \leq 4$ são denotados por G_i, com respectivamente n_i vértices. De acordo com a partição proposta, o grafo é rotulado de forma que a matriz laplaciana de G pode ser escrita na forma de blocos da seguinte maneira

$$L(G) = \begin{bmatrix} d(v_1) & \mathbf{-1} & \mathbf{0} & 0 \\ \mathbf{-1}^T & L_{22} + I & L_{23} & \mathbf{-1} \\ \mathbf{0}^T & L_{32} & L_{33} + I & \mathbf{-1} \\ 0^T & \mathbf{-1}^T & \mathbf{-1}^T & d(v) \end{bmatrix},$$

onde $L_{ii} = L(G_i), \forall i \ 2 \leq i \leq 3$. O subgrafo obtido de G pela remoção de v, $G \setminus v$, tem a matriz laplaciana dada por

$$L(G \setminus v) = \begin{bmatrix} d(v_1) & \mathbf{-1} & \mathbf{0} \\ \mathbf{-1}^T & L_{22} & L_{23} \\ \mathbf{0}^T & L_{32} & L_{33} \end{bmatrix}.$$

A partir da caracterização da matriz laplaciana, obtivemos os dois principais resultados dessa dissertação.

Proposição IV.3. Sejam $G \in \Im_{n,n-2}$ e v o vértice de grau $(n-2)$, tal que $n_2 \leq n_3 + 1$. Se $G = G_2 \vee (G_1 + (G_3 \vee G_4))$, então $\phi(v) = 0$ e $a(G) = \delta(G) = n_2$.

Demonstração:

Suponhamos que $G = G_2 \vee (G_1 + (G_3 \vee G_4))$ e que $n_2 \leq n_3 + 1$. Logo, $G \setminus v = G_2 \vee (G_1 + G_3)$. Pela Proposição I.28, temos que os autovalores de $L(G)$ e $L(G \setminus v)$ são

$\{n_2 + n_3 + 2,\ \mu_1(G_2) + n_3 + 2,\ \ldots\ ,\ \mu_{n_2-1}(G_2) + n_3 + 2,\ \mu_1(G_3 \vee G_4) + n_2, \ldots,$
$\mu_{n_3}(G_3 \vee G_4) + n_2, n_2, 0\} =$
$\{n_2 + n_3 + 2, \mu_1(G_2) + n_3 + 2, \ldots, \mu_{n_2-1}(G_2) + n_3 + 2, n_3 + n_2 + 1, \mu_1(G_3) + n_2 + 1, \ldots,$
$\mu_{n_3-1}(G_3) + n_2 + 1, n_2, 0\}$ e
$\{n_2 + n_3 + 1, \mu_1(G_2) + n_3 + 1, \ldots, \mu_{n_2-1}(G_2) + n_3 + 1, \mu_1(G_3) + n_2, \ldots, \mu_{n_3-1}(G_3) + n_2, n_2, 0\}$,
respectivamente.

Como por hipótese $n_2 \leq n_3 + 1$, tem-se que $n_2 < \mu_{n_2-1}(G_2) + n_3 + 2$ e $n_2 \leq \mu_{n_2-1}(G_2) + n_3 + 1$. Portanto, $a(G) = n_2$ e $a(G \setminus v) = n_2$ e, conseqüentemente, $\phi(v) = 0$. Além disso, como $d(v_1) = n_2$, conclui-se que $a(G) = \delta(G) = n_2$.■

Proposição IV.4. Sejam $G \in \Im_{n,n-2}$ tal que $n_2 \leq n_3 + 1$. Se $G = G_2 \vee (G_1 + (G_3 \vee G_4))$ e

$$x = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \hline -1 \\ \hline n-1-a(G) \\ \vdots \\ \hline -1 \\ \hline n-1-a(G) \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} a(G) \\ \left.\begin{matrix} \\ \\ \\ \\ \\ \\ \end{matrix}\right\} n-1-a(G) \end{matrix} \quad e\ \bar{x} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \hline -1 \\ \hline n-2-a(G) \\ \vdots \\ \hline -1 \\ \hline n-2-a(G) \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} a(G) \\ \left.\begin{matrix} \\ \\ \\ \\ \\ \\ \end{matrix}\right\} n-2-a(G) \end{matrix},$$

então x e $\overline{x}$ são os vetores de Fiedler de $L(G)$ e $L(G \setminus v)$, respectivamente.
Demonstração:

Suponhamos que $G \in \Im_n$ e $G = G_2 \vee (G_1 + (G_3 \vee G_4))$ tal que $n_2 \leq n_3 + 1$. Denotamos por **J** a matriz cujas entradas são todas iguais a 1. Então, pela Proposição IV.3, temos que

$$L(G) \cdot x = \begin{bmatrix} a(G) & \textbf{-1} & \textbf{0} & 0 \\ \textbf{-1}^T & L_{22} + I & \textbf{-J} & \textbf{-1} \\ \textbf{0}^T & \textbf{-J}^T & L_{33} + I & \textbf{-1} \\ 0^T & \textbf{-1}^T & \textbf{-1}^T & (n-2) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \hline -1 \\ \hline n-1-a(G) \\ \vdots \\ \hline -1 \\ \hline n-1-a(G) \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} a(G) \\ \left.\begin{matrix} \\ \\ \\ \\ \\ \\ \end{matrix}\right\} n-1-a(G) \end{matrix} =$$

$$\begin{bmatrix} a(G) \\ 0 \\ \vdots \\ 0 \\ \frac{-a(G)}{n-1-a(G)} \\ \vdots \\ \frac{-a(G)}{n-1-a(G)} \end{bmatrix} = a(G) \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \frac{-1}{n-1-a(G)} \\ \vdots \\ \frac{-1}{n-1-a(G)} \end{bmatrix} = a(G) \cdot x.$$

Logo, x é um vetor de Fiedler de $L(G)$. De maneira similar, mostra-se que $L(G \setminus v) \cdot \bar{x} = a(G \setminus v) \cdot \bar{x}$.■

Com base nos testes computacionais, observamos que em todos os casos em que G possui um vértice de grau $(n-2)$, que gera $\phi(v) = 0$, G é um *join* e o vetor de Fiedler tem um comportamento padrão, por isso suspeitamos fortemente que a recíproca das Proposições IV.3 e IV.4 sejam verdadeiras e enunciamos estes resultados em forma de conjecturas.

Conjectura IV.5. Seja $G \in \Im_n$ tal que $n_2 \leq n_3 + 1$. Então, $\phi(v) = 0$ se e somente se $G = G_2 \vee (G_1 + (G_3 \vee G_4))$.

Demonstração:

($\Rightarrow$) Em aberto.

($\Leftarrow$) Proposição IV.3.

Conjectura IV.6. Seja $G \in \Im_n$ tal que $n_2 \leq n_3 + 1$. Então, $\phi(v) = 0$ se e somente se os vetores de Fiedler x e $\overline{x}$ de $L(G)$ e $L(G \setminus v)$, respectivamente, são da forma

$$x = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \frac{-1}{n-1-a(G)} \\ \vdots \\ \frac{-1}{n-1-a(G)} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} a(G) \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} n-1-a(G) \end{matrix} \quad e\ \bar{x} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \frac{-1}{n-2-a(G)} \\ \vdots \\ \frac{-1}{n-2-a(G)} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} a(G) \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} n-2-a(G) \end{matrix}.$$

De acordo com a Proposição IV.3, temos que se $n_2 \leq n_3 + 1$ e $G = G_2 \vee (G_1 + (G_3 \vee G_4))$,

então $\phi(v) = 0$. Entretanto, outros resultados decorrem do fato de que $G = G_2 \vee (G_1 + (G_3 \vee G_4))$. Como pode ser visto na Tabela IV.5, quando consideramos $n_2 > n_3 + 2$, outros casos podem ser gerados inclusive $\phi(v) = 0$.

$a(G)$	$a(G \setminus v)$	$\phi(v)$
$a(G_2) + n_3 + 2$ ou n_2	$a(G_2) + n_3 + 1$ ou n_2	1 ou $a(G_2) + n_3 + 2 - n_2$
	$a(G_2) + n_3 + 1$ ou n_2	$n_2 - a(G_2) - n_3 - 1$ ou 0

Tabela IV.5: Outros resultados para $\phi(v)$

Através da Tabela IV.5, podemos observar que o valor da função $\phi(v)$ depende da conectividade algébrica de G_2 em dois casos e, portanto, de acordo com [33] pode assumir qualquer valor menor ou igual a 1. Além disso, podemos observar que os grafos que geram $\phi(v) = 0$ são aqueles em que $a(G_2) = n_2 - n_3 - 2$ ou $a(G_2) = n_2 - n_3 - 1$, onde $n_2 > n_3 + 2$.

Conclusões e Trabalhos Futuros

Neste trabalho, solucionamos parcialmente a questão levantada por Kirkland [33]. Com o objetivo de solucionar esta questão, analisamos o comportamento da função de perturbação $\phi(v)$ dentro de algumas famílias de grafos e em grafos resultantes de operações, onde verificamos a ocorrência tanto da função $\phi(v) = 1$ quanto $\phi(v) = 0$, vide Tabela II.1. A partir dessa análise, decidimos investigar a função $\phi(v) = 0$, pois a função de perturbação nula ainda não havia sido caracterizada. Uma das contribuições de nossa pesquisa nessa primeira fase, foi o cálculo do intervalo para a conectividade algébrica dos grafos resultantes da operação ponte de ligação entre completos. Na segunda fase de nossa pesquisa, verificamos dentro das famílias de grafos estudadas quais possuíam vértices de grau $(n-2)$ e, se possuíam, que função de perturbação geravam. Essa análise foi estendida a todos os grafos conexos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$ dos quais 505 apresentavam vértices de grau $(n-2)$. Desse total, 46 grafos geravam $\phi(v) = 0$. Analisamos a estrutura desses grafos e concluímos que eles necessariamente são formados por um *join* de 4 grafos. Então, rotulamos e particionamos estes grafos de forma que suas matrizes laplacianas fossem divididas em blocos. A partir da caracterização da matriz laplaciana, conseguimos provar o nosso principal resultado teórico, apresentado na Proposição IV.3, onde provamos que se $G = G_2 \vee (G_1 + (G_3 \vee G_4))$, tal que $n_2 \leq n_3 + 1$, então $\phi(v) = 0$ e, portanto, $a(G) = \delta(G) = n_2$. Desse resultado decorreram a Proposição IV.4, onde caracterizamos os vetores de Fiedler para estes grafos, e as Conjecturas IV.5 e IV.6, que acreditamos serem verdadeiras, levando-se em conta o que foi observado nos experimentos computacionais. Vale ressaltar que os resultados obtidos foram de fundamental importância para descrevermos os grafos com $\phi(v) = 0$, onde $d(v) = n - 2$, porém ainda há muito a ser investigado no que se refere à abrangência da função $\phi(v) = 0$. Desta forma, sugerimos como trabalhos futuros a demonstração das conjecturas apresentadas no Capítulo IV, assim como, a caracterização dos grafos $G = G_2 \vee (G_1 + (G_3 \vee G_4))$ nos quais $a(G_2) = n_2 - n_3 - 2$ ou $a(G_2) = n_2 - n_3 - 1$, onde $n_2 > n_3 + 1$.

Referências Bibliográficas

[1] A. E. Brouwer, W. H. Haemers. **Spectra of Graphs**, Spring New York, (2012).

[2] A. K. Lal, K. L. Patra, B. K. Sahoo. **Algebraic Connectivity of Connected Graphs with Fixed Number of Pendant Vertices**, Graphs and Combinatorics, Issue 2, v. 27, pp. 215-229, (2011).

[3] A. T. Amim, S. L. Hakini. **Graphs with Given Connectivity and Independence Number or Networks with Given Measures of Vulnerability and Survivability**, IEEE Trans. Circuit Theory, v. CT-20, pp. 2-10, (1973).

[4] C. Godsil, G. Royle. **Algebraic Graph Theory**, Graduate Texts in Mathematics 207, Springer Verlag NY, (2001).

[5] C. S. Oliveira. **Laplaciano de Grafos e Vulnerabilidade em Redes**, Tese de D.Sc., COPPE/UFRJ, (2003).

[6] D. Cvetković. **Graphs and their Spectra**, Univ. Beograd, Publ. Elektrotehn, Fak., Ser. Mat. Fiz., vol. 354, pp. 1-50, (1971).

[7] D. Cvetković, M. Aouchiche, F. Bell, *et al*. **Variable Neighborhood Search for Extremal Graphs. 16.Some Conjectures Related to the Largest Eigenvalue of a Graph**, European Journal of Operational Research, issue 3, v. 191, pp. 661-676, (2008).

[8] D. Cvetković, M. Doob, H. Sachs. **Spectra of Graphs-Theory and Applications**, 3 ed. New York, Academic Press, (1979).

[9] D. Cvetković, M. Doob, H. Sachs. **Spectra of Graphs**, 1 ed. New York, Academic Press, (1995).

[10] D. Cvetković, M. Doob, I. Gutman, A. Torgãsev. **Recent Results in the Theory of Graph Spectra**, North Holland, Amsterdam, (1988).

[11] D. Cvetković, P. Rowlinson, S. Simic. **An Introduction to the Theory of Graph Spectra**, London Mathematical Society Student Texts, (2009).

[12] F. Harary. **Graph Theory**, Addison Wesley, (1969).

[13] F. Harary. **The Maximum Connectivity of a Graph**, Proc. Nat. Acad. Sci. U.S.A., v. 48, pp. 1142-1146, (1962).

[14] F. T. Boesch, A. P. Felze. **A General Class of Invulnerable Graphs, Networks**, v. 2, pp. 261-283, (1972).

[15] F. T. Boesch, R. E. Thomas. **On graphs of Invulnerable Communications Nets**, IEEE Trans. Circuit Theory, v. CT-17, pp. 183-192, (1970).

[16] G. Caporossi, P. Hansen. **Variable Neighborhood Search for Extremal Graphs: The Autographix System**, Discrete Mathematics, v. 212, pp. 29-34, (2000).

[17] J. G. Michaels, K. H. Rosen. **Applications of Discrete Mathematics**, Mc Graw-Hill, (1992).

[18] J.M.S. Simões Pereira. **Matemática Discreta: Grafos, Redes, Aplicações**, Ed. Luz da Vida, (2009).

[19] J.-M. Guo. **On the Second Largest Laplacian Eigenvalue of Trees**, Linear Algebra and its Applications, v. 404, pp. 251-261, (2005).

[20] J. W. Eaton. **GNU Octave - A High-Level Interactive Language for Numerical Computation**, version $3.2.2$, (2009).

[21] K. L. Patra, A. K. Lal. **The Effect on the Algebraic Connectivity of a Tree by Grafting or Collapsing of Edges**, Linear Algebra and its Applications, v. 428, pp. 855-864, (2008).

[22] M. Fiedler. **Algebraic Connectivity of Graphs**, Czechoslovak Mathematical Journal, v. 23 (98), pp. 298-305, (1973).

[23] M. N. S. Swamy, K. Thwlasiraman . **Graphs, Networks and Algorithms**, (1981).

[24] N. Biggs. **Algebraic Graph Theory**, 2 ed. Great Britain, Cambridge University Press, (1993).

[25] N. M. M. de Abreu. **Old and New Results on Algebraic Connectivity of Graphs, Linear Algebra and its Applications**, v. 423, pp. 53-73, (2007).

[26] N. M. M. de Abreu, R. R. Del-Vecchio, C. T. M. Vinagre. **Introdução à Teoria Espectral de Grafos com Aplicações**, Sociedade Brasileira de Matemática Aplicada e Computacional, São Carlos-SP, Brasil, (2007).

[27] R. Diestel. **Graph Theory**, Springer-Verlag, (1996).

[28] R. Grone, R. Merris. **Ordering Trees by Algebraic connectivity**, Graphs and Combinatorics, v. 6, pp. 229-237, (1990).

[29] R. Merris. **Graph Theory**, Canada, Wiley-Interscience Publication, (2001).

[30] R. Merris. **Laplacian Matrices of Graphs: A Survey, Linear Algebra and its Applications**, v. 197/198, pp. $143 - 176$, (1994).

[31] S. K. Simić, B. Zhou. **Indices of Trees with a Prescribed Diameter**, Applicable Analysis and Discrete Mathematics, v. 1, pp. 446454, (2007).

[32] S. Kirkland. **Algebraic Connectivity**, in: Leslie Hogben (Ed.), Handbook of Linear Algebra, Chapman Hall/CRC, Boca Raton, pp. 36-1-36-12, (2007).

[33] S. Kirkland. **Algebraic Connectivity for Vertex-deleted Subgraphs, and a Notion of Vertex Centrality**, Discrete Mathematics, issue 4, v. 310, pp. 911-921, (2010).

[34] S. Kirkland, C. S. Oliveira, C. M. Justel. **On Algebraic Connectivity Augmentation**, v. 435, pp. 2347-2356, (2011).

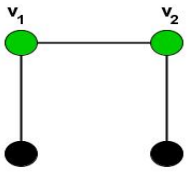
[35] S. M. Fallat; S. Kirkland; S. Pati. **Maximizing Algebraic Connectivity over Unicyclic Graphs**, Linear and Multilinear Algebra, Issue 3, v. 51, pp. 221-241, (2003).

[36] V. Nikiforov, C. S. Oliveira, L. S. de Lima, N. M. M. de Abreu. **The Smallest Eigenvalue of Signless Laplacian**, Linear Algebra and its Applications, Issue 10, v. 435, pp. 2570-2584, (2011).

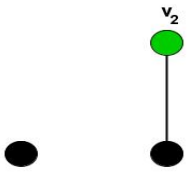
[37] X. Y. Yuan; J. Shao; L. Zhang. **The Six Classes of Trees with the Largest Algebraic Connectivity**, Discrete Applied Mathematics archive, Issue 5, v. 156, pp. 757-769, (2008).

Apêndice A - Grafos que possuem vértices de grau $(n-2)$ e seus subgrafos

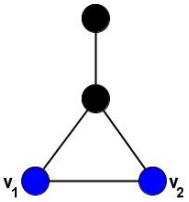
Os grafos apresentados abaixo possuem pelo menos um vértice de grau $(n-2)$, cuja remoção gera uma função $\phi(v)$. Estes vértices foram rotulados e destacados nas cores azul e verde. Os vértices de cor azul geram $\phi(v) = 0$, enquanto que a remoção dos vértices de cor verde geram $\phi(v) \neq 0$. Os vértices rotulados e não removidos são vértices isomorfos e, portanto, foram desconsiderados.



A.1.1: G_1



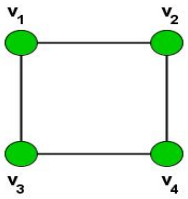
A.1.2: $G_1 \setminus v_1$



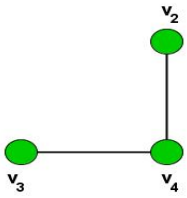
A.1.3: G_2



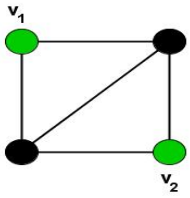
A.1.4: $G_2 \setminus v_1$



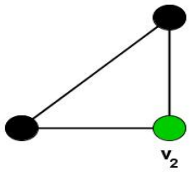
A.1.5: G_3



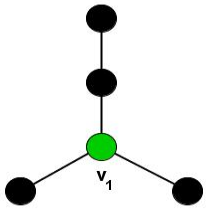
A.1.6: $G_3 \setminus v_1$



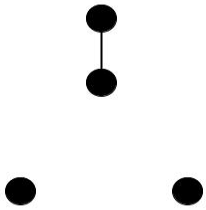
A.1.7: G_4



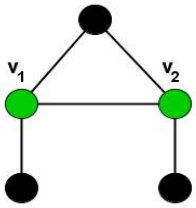
A.1.8: $G_4 \setminus v_1$



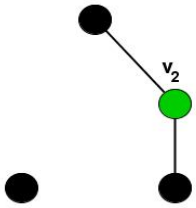
A.1.9: G_5



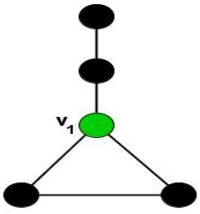
A.1.10: $G_5 \setminus v_1$



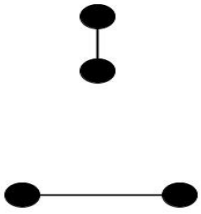
A.1.11: G_6



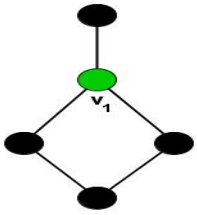
A.1.12: $G_6 \setminus v_1$



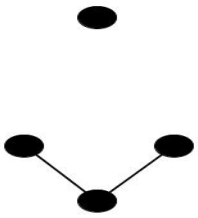
A.1.13: G_7



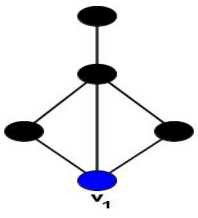
A.1.14: $G_7 \setminus v_1$



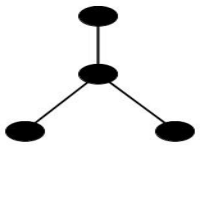
A.1.15: G_8



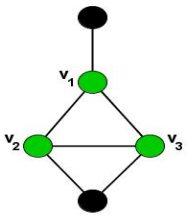
A.1.16: $G_8 \setminus v_1$



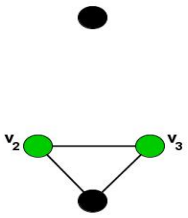
A.1.17: G_9



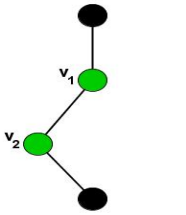
A.1.18: $G_9 \setminus v_1$



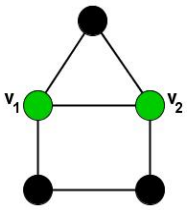
A.1.19: G_{10}



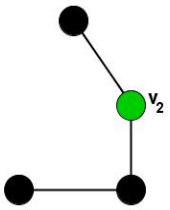
A.1.20: $G_{10} \setminus v_1$



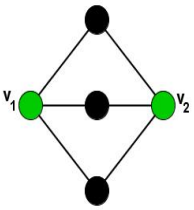
A.1.21: $G_{10} \setminus v_3$



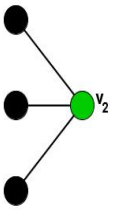
A.1.22: G_{11}



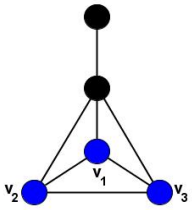
A.1.23: $G_{11} \setminus v_1$



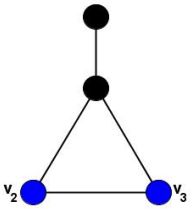
A.1.24: G_{12}



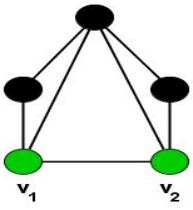
A.1.25: $G_{12} \setminus v_1$



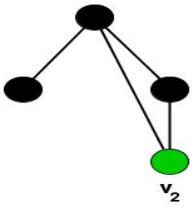
A.1.26: G_{13}



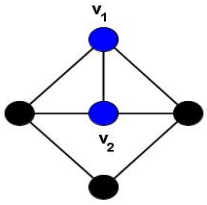
A.1.27: $G_{13} \setminus v_1$



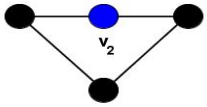
A.1.28: G_{14}



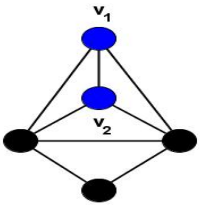
A.1.29: $G_{14} \setminus v_1$



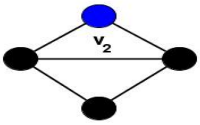
A.1.30: G_{15}



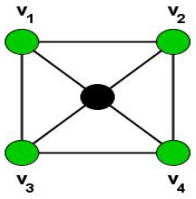
A.1.31: $G_{15} \setminus v_1$



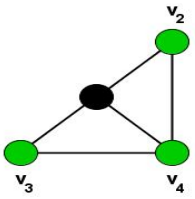
A.1.32: G_{16}



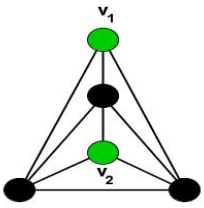
A.1.33: $G_{16} \setminus v_1$



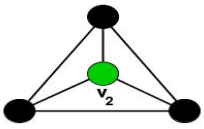
A.1.34: G_{17}



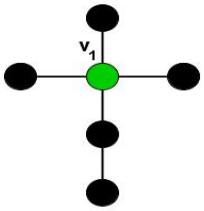
A.1.35: $G_{17} \setminus v_1$



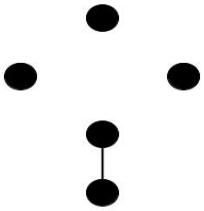
A.1.36: G_{18}



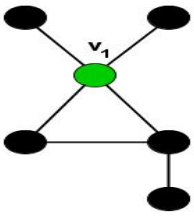
A.1.37: $G_{18} \setminus v_1$



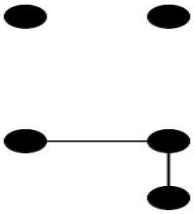
A.1.38: G_{19}



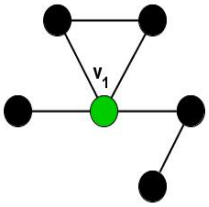
A.1.39: $G_{19} \setminus v_1$



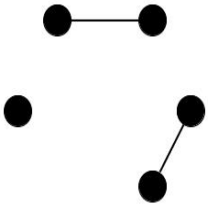
A.1.40: G_{20}



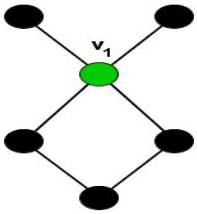
A.1.41: $G_{20} \setminus v_1$



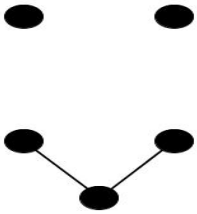
A.1.42: G_{21}



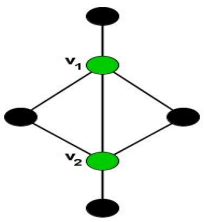
A.1.43: $G_{21} \setminus v_1$



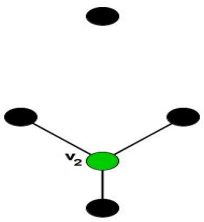
A.1.44: G_{22}



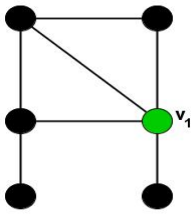
A.1.45: $G_{22} \setminus v_1$



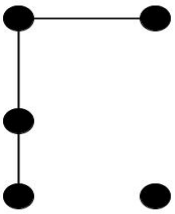
A.1.46: G_{23}



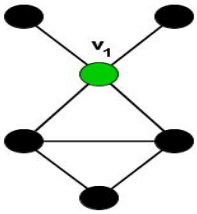
A.1.47: $G_{23} \setminus v_1$



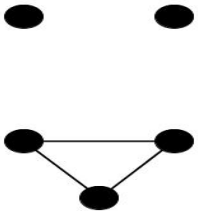
A.1.48: G_{24}



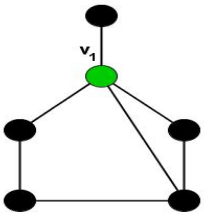
A.1.49: $G_{24} \setminus v_1$



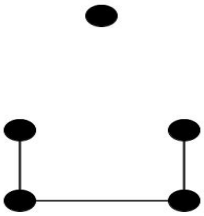
A.1.50: G_{25}



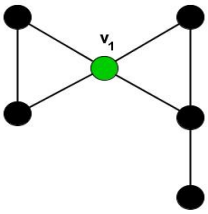
A.1.51: $G_{25} \setminus v_1$



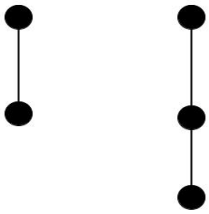
A.1.52: G_{26}



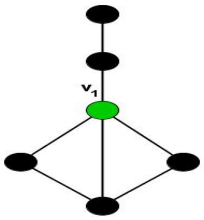
A.1.53: $G_{26} \setminus v_1$



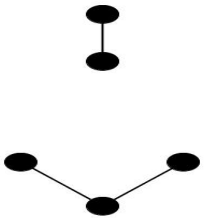
A.1.54: G_{27}



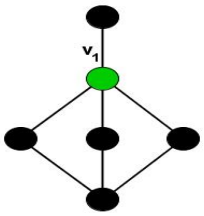
A.1.55: $G_{27} \setminus v_1$



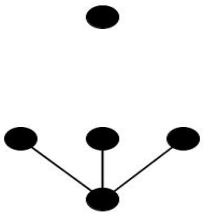
A.1.56: G_{28}



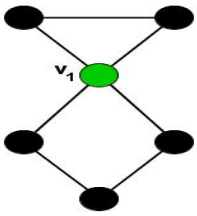
A.1.57: $G_{28} \setminus v_1$



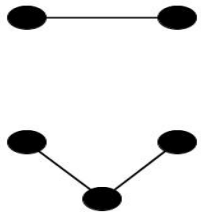
A.1.58: G_{29}



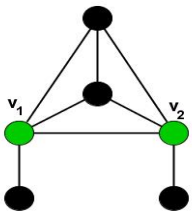
A.1.59: $G_{29} \setminus v_1$



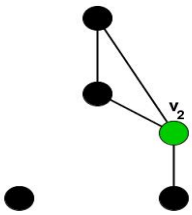
A.1.60: G_{30}



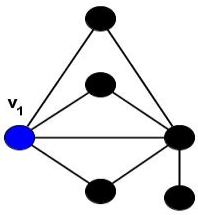
A.1.61: $G_{30} \setminus v_1$



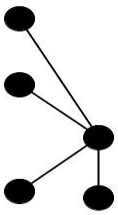
A.1.62: G_{31}



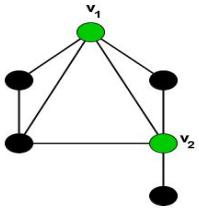
A.1.63: $G_{31} \setminus v_1$



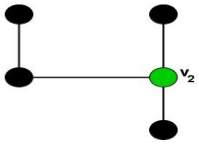
A.1.64: G_{32}



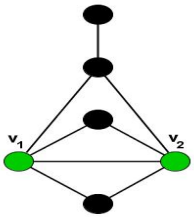
A.1.65: $G_{32} \setminus v_1$



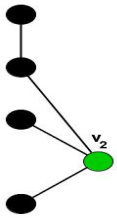
A.1.66: G_{33}



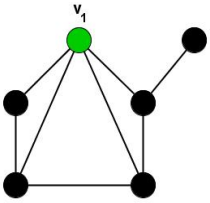
A.1.67: $G_{33} \setminus v_1$



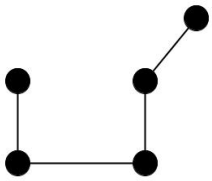
A.1.68: G_{34}



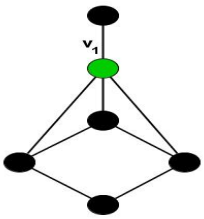
A.1.69: $G_{34} \setminus v_1$



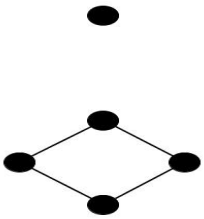
A.1.70: G_{35}



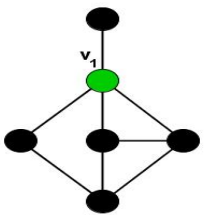
A.1.71: $G_{35} \setminus v_1$



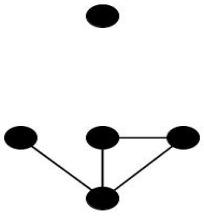
A.1.72: G_{36}



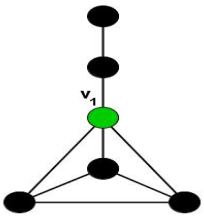
A.1.73: $G_{36} \setminus v_1$



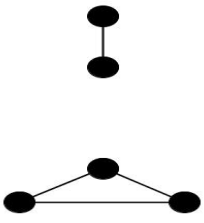
A.1.74: G_{37}



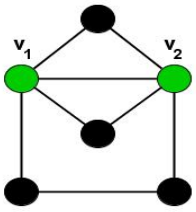
A.1.75: $G_{37} \setminus v_1$



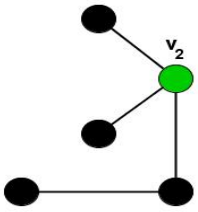
A.1.76: G_{38}



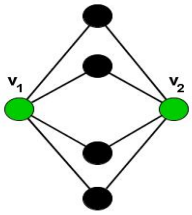
A.1.77: $G_{38} \setminus v_1$



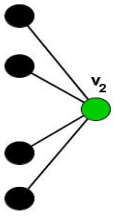
A.1.78: G_{39}



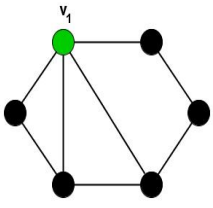
A.1.79: $G_{39} \setminus v_1$



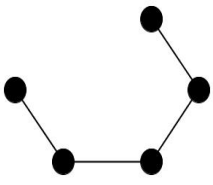
A.1.80: G_{40}



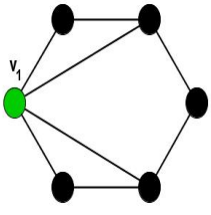
A.1.81: $G_{40} \setminus v_1$



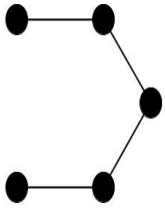
A.1.82: G_{41}



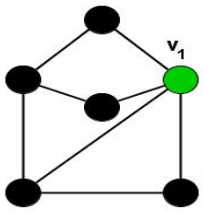
A.1.83: $G_{41} \setminus v_1$



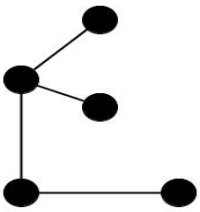
A.1.84: G_{42}



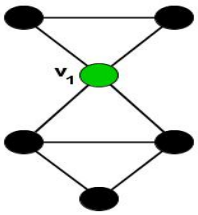
A.1.85: $G_{42} \setminus v_1$



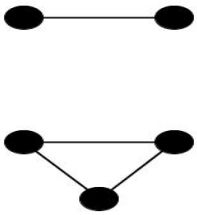
A.1.86: G_{43}



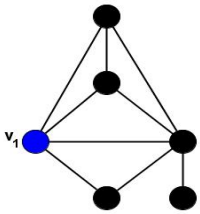
A.1.87: $G_{43} \setminus v_1$



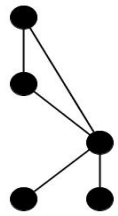
A.1.88: G_{44}



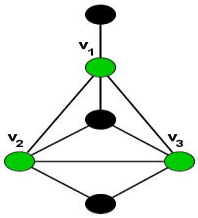
A.1.89: $G_{44} \setminus v_1$



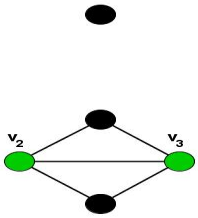
A.1.90: G_{45}



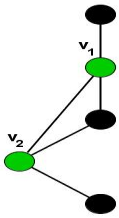
A.1.91: $G_{45} \setminus v_1$



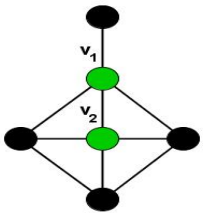
A.1.92: G_{46}



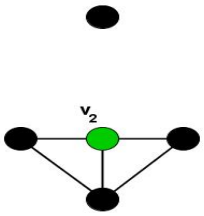
A.1.93: $G_{46} \setminus v_1$



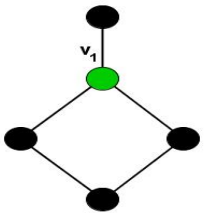
A.1.94: $G_{46} \setminus v_3$



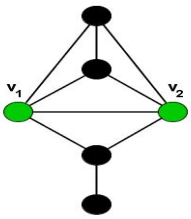
A.1.95: G_{47}



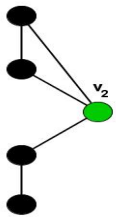
A.1.96: $G_{47} \setminus v_1$



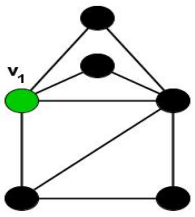
A.1.97: $G_{47} \setminus v_2$



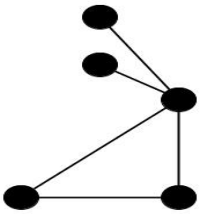
A.1.98: G_{48}



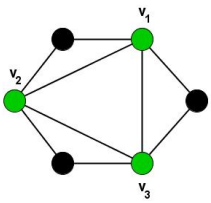
A.1.99: $G_{48} \setminus v_1$



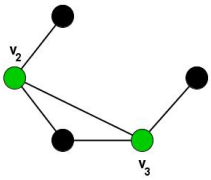
A.1.100: G_{49}



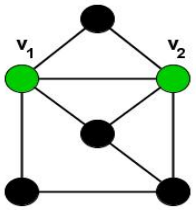
A.1.101: $G_{49} \setminus v_1$



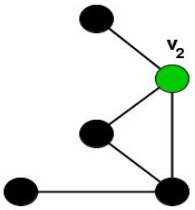
A.1.102: G_{50}



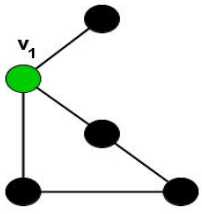
A.1.103: $G_{50} \setminus v_1$



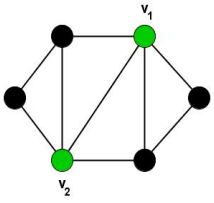
A.1.104: G_{51}



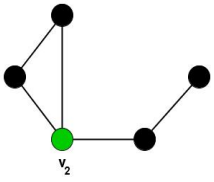
A.1.105: $G_{51} \setminus v_1$



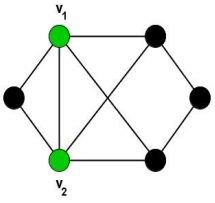
A.1.106: $G_{51} \setminus v_2$



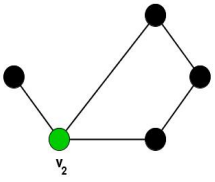
A.1.107: G_{52}



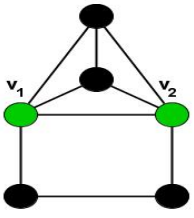
A.1.108: $G_{52} \setminus v_1$



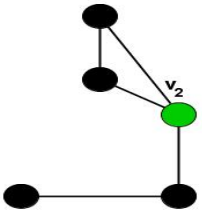
A.1.109: G_{53}



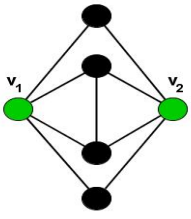
A.1.110: $G_{53} \setminus v_1$



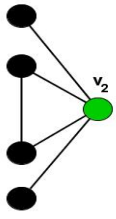
A.1.111: G_{54}



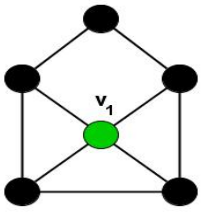
A.1.112: $G_{54} \setminus v_1$



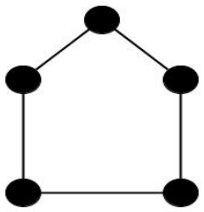
A.1.113: G_{55}



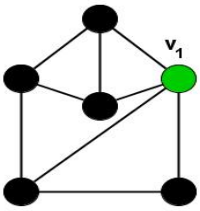
A.1.114: $G_{55} \setminus v_1$



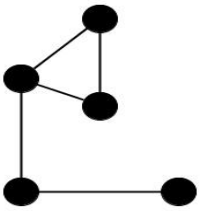
A.1.115: G_{56}



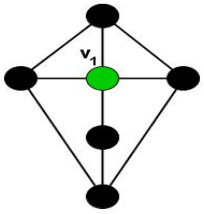
A.1.116: $G_{56} \setminus v_1$



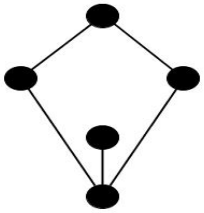
A.1.117: G_{57}



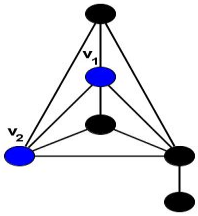
A.1.118: $G_{57} \setminus v_1$



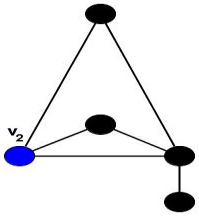
A.1.119: G_{58}



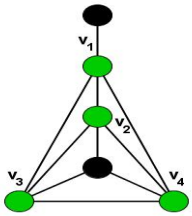
A.1.120: $G_{58} \setminus v_1$



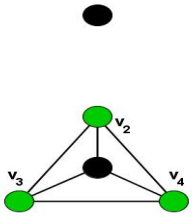
A.1.121: G_{59}



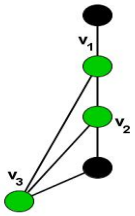
A.1.122: $G_{59} \setminus v_1$



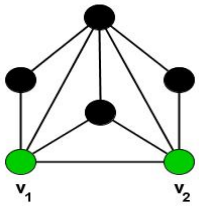
A.1.123: G_{60}



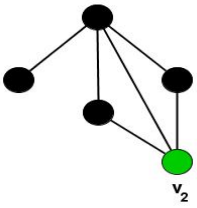
A.1.124: $G_{60} \setminus v_1$



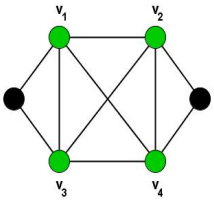
A.1.125: $G_{60} \setminus v_4$



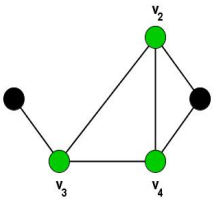
A.1.126: G_{61}



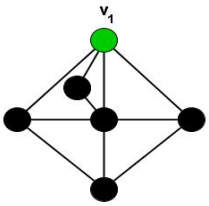
A.1.127: $G_{61} \setminus v_1$



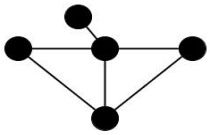
A.1.128: G_{62}



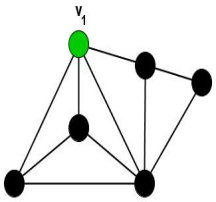
A.1.129: $G_{62} \setminus v_1$



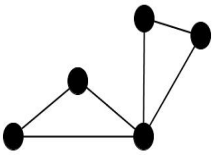
A.1.130: G_{63}



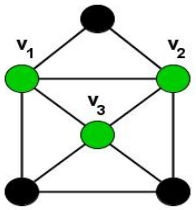
A.1.131: $G_{63} \setminus v_1$



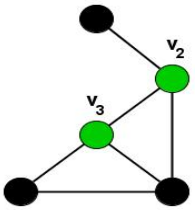
A.1.132: G_{64}



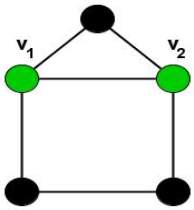
A.1.133: $G_{64} \setminus v_1$



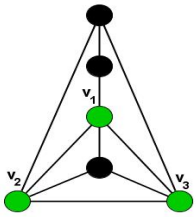
A.1.134: G_{65}



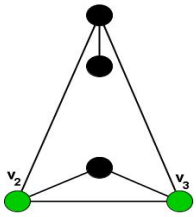
A.1.135: $G_{65} \setminus v_1$



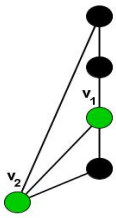
A.1.136: $G_{65} \setminus v_3$



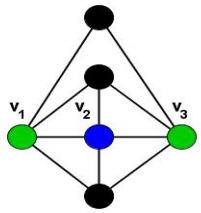
A.1.137: G_{66}



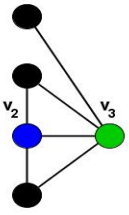
A.1.138: $G_{66} \setminus v_1$



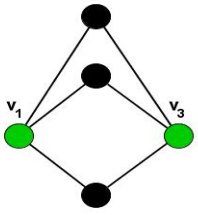
A.1.139: $G_{66} \setminus v_3$



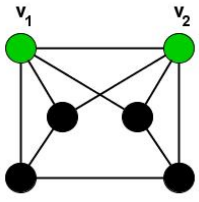
A.1.140: G_{67}



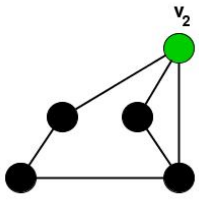
A.1.141: $G_{67} \setminus v_1$



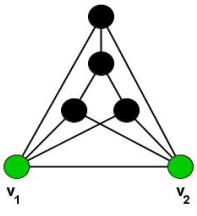
A.1.142: $G_{67} \setminus v_2$



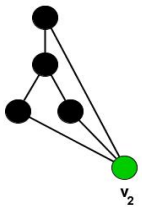
A.1.143: G_{68}



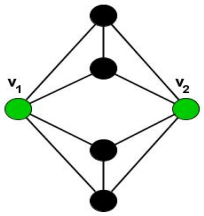
A.1.144: $G_{68} \setminus v_1$



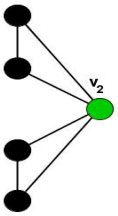
A.1.145: G_{69}



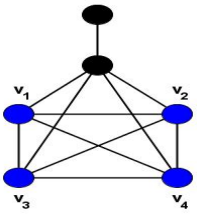
A.1.146: $G_{69} \setminus v_1$



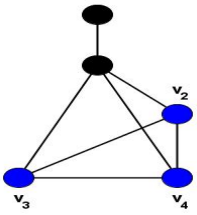
A.1.147: G_{70}



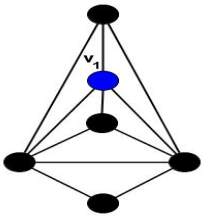
A.1.148: $G_{70} \setminus v_1$



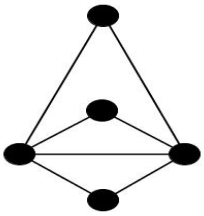
A.1.149: G_{71}



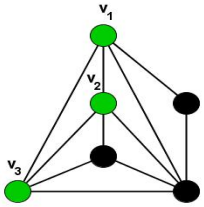
A.1.150: $G_{71} \setminus v_1$



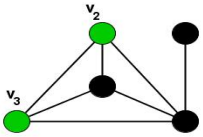
A.1.151: G_{72}



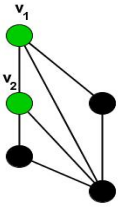
A.1.152: $G_{72} \setminus v_1$



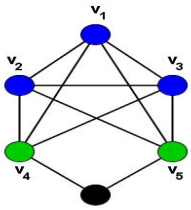
A.1.153: G_{73}



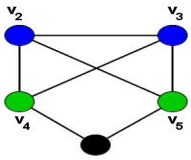
A.1.154: $G_{73} \setminus v_1$



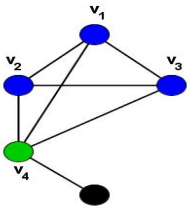
A.1.155: $G_{73} \setminus v_3$



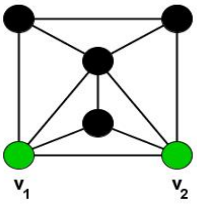
A.1.156: G_{74}



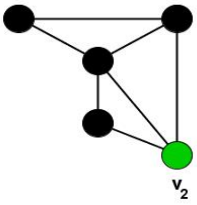
A.1.157: $G_{74} \setminus v_1$



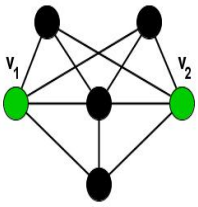
A.1.158: $G_{74} \setminus v_5$



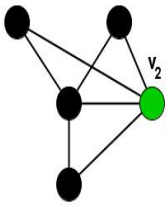
A.1.159: G_{75}



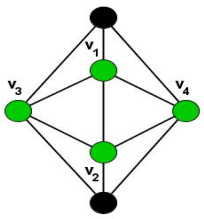
A.1.160: $G_{75} \setminus v_1$



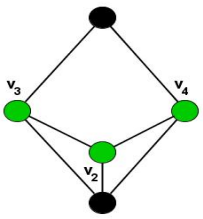
A.1.161: G_{76}



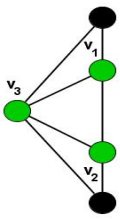
A.1.162: $G_{76} \setminus v_1$



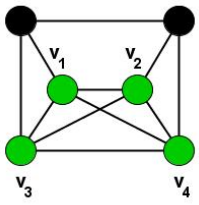
A.1.163: G_{77}



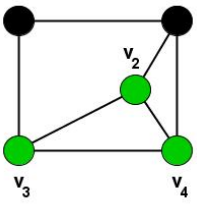
A.1.164: $G_{77} \setminus v_1$



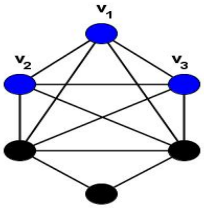
A.1.165: $G_{77} \setminus v_4$



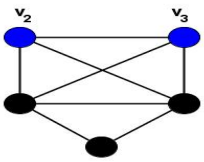
A.1.166: G_{78}



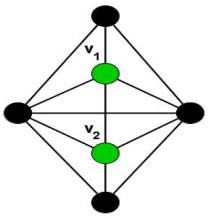
A.1.167: $G_{78} \setminus v_1$



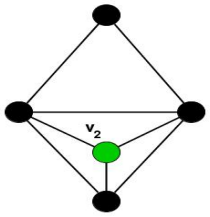
A.1.168: G_{79}



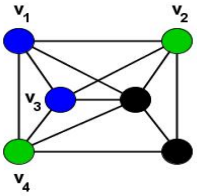
A.1.169: $G_{79} \setminus v_1$



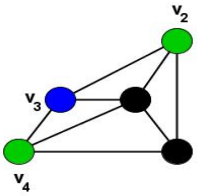
A.1.170: G_{80}



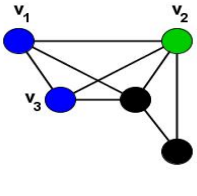
A.1.171: $G_{80} \setminus v_1$



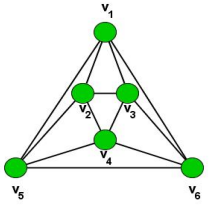
A.1.172: G_{81}



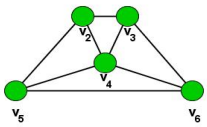
A.1.173: $G_{81} \setminus v_1$



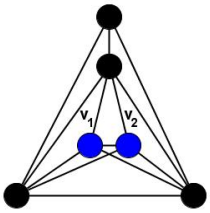
A.1.174: $G_{81} \setminus v_4$



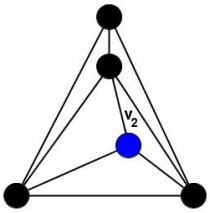
A.1.175: G_{82}



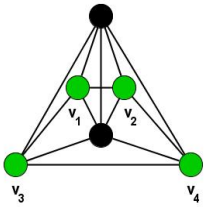
A.1.176: $G_{82} \setminus v_1$



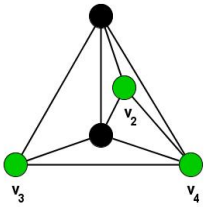
A.1.177: G_{83}



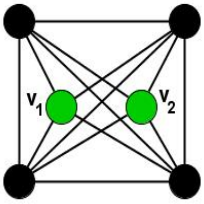
A.1.178: $G_{83} \setminus v_1$



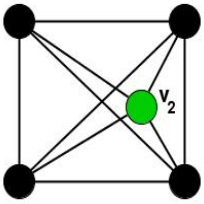
A.1.179: G_{84}



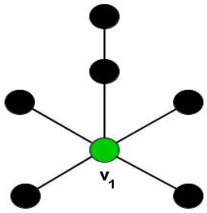
A.1.180: $G_{84} \setminus v_1$



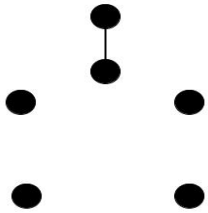
A.1.181: G_{85}



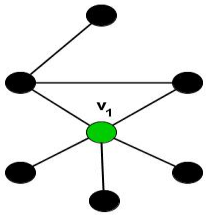
A.1.182: $G_{85} \setminus v_1$



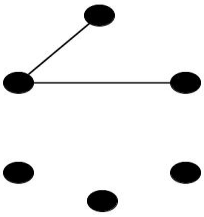
A.1.183: G_{86}



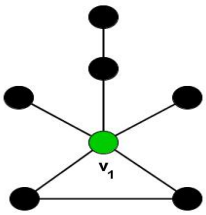
A.1.184: $G_{86} \setminus v_1$



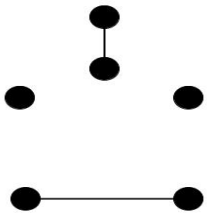
A.1.185: G_{87}



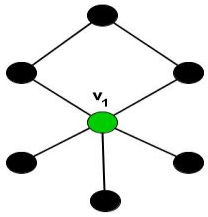
A.1.186: $G_{87} \setminus v_1$



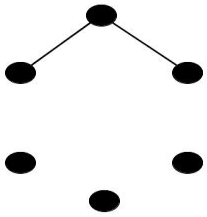
A.1.187: G_{88}



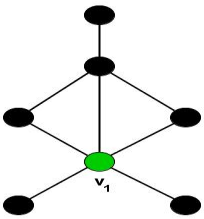
A.1.188: $G_{88} \setminus v_1$



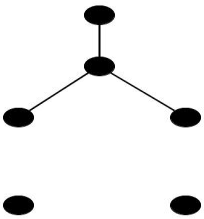
A.1.189: G_{89}



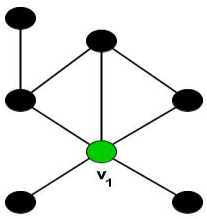
A.1.190: $G_{89} \setminus v_1$



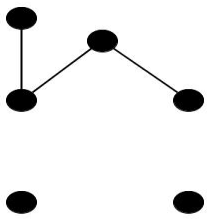
A.1.191: G_{90}



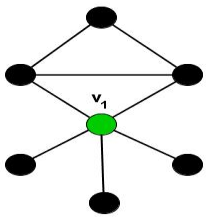
A.1.192: $G_{90} \setminus v_1$



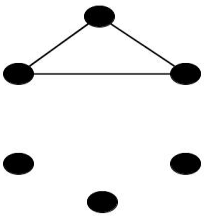
A.1.193: G_{91}



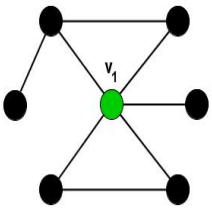
A.1.194: $G_{91} \setminus v_1$



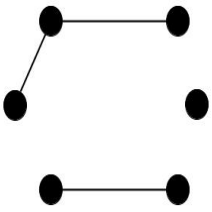
A.1.195: G_{92}



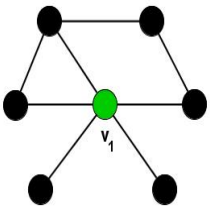
A.1.196: $G_{92} \setminus v_1$



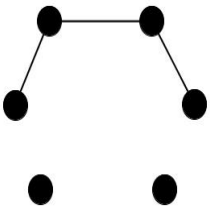
A.1.197: G_{93}



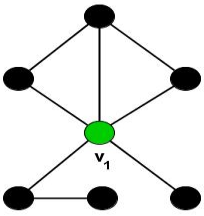
A.1.198: $G_{93} \setminus v_1$



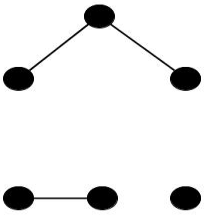
A.1.199: G_{94}



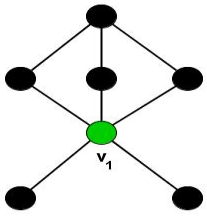
A.1.200: $G_{94} \setminus v_1$



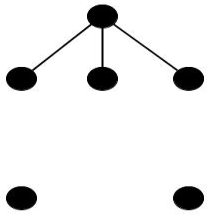
A.1.201: G_{95}



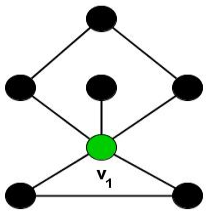
A.1.202: $G_{95} \setminus v_1$



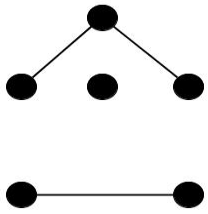
A.1.203: G_{96}



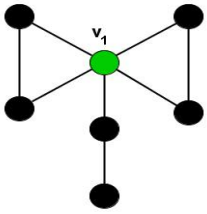
A.1.204: $G_{96} \setminus v_1$



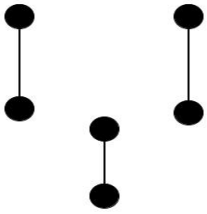
A.1.205: G_{97}



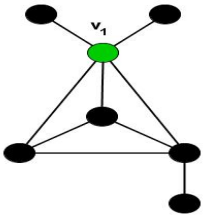
A.1.206: $G_{97} \setminus v_1$



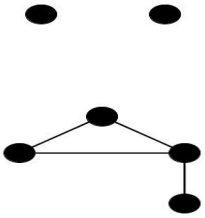
A.1.207: G_{98}



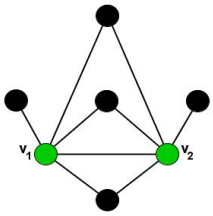
A.1.208: $G_{98} \setminus v_1$



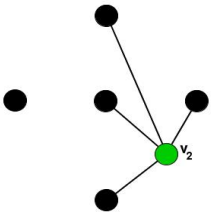
A.1.209: G_{99}



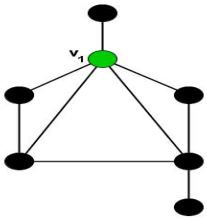
A.1.210: $G_{99} \setminus v_1$



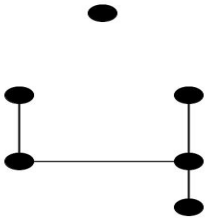
A.1.211: G_{100}



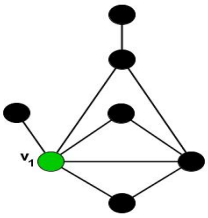
A.1.212: $G_{100} \setminus v_1$



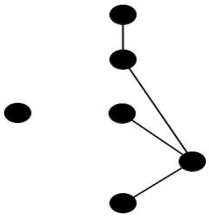
A.1.213: G_{101}



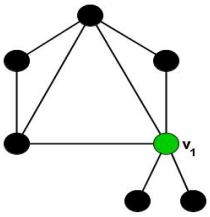
A.1.214: $G_{101} \setminus v_1$



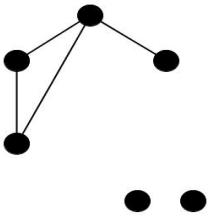
A.1.215: G_{102}



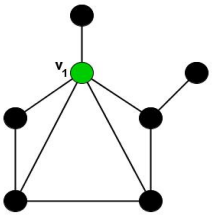
A.1.216: $G_{102} \setminus v_1$



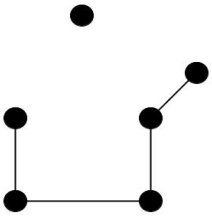
A.1.217: G_{103}



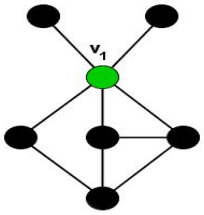
A.1.218: $G_{103} \setminus v_1$



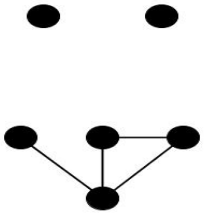
A.1.219: G_{104}



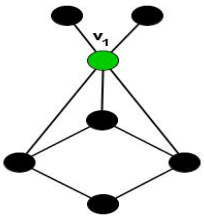
A.1.220: $G_{104} \setminus v_1$



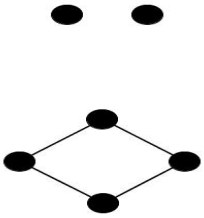
A.1.221: G_{105}



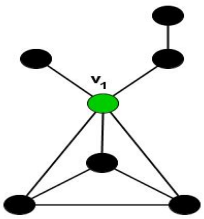
A.1.222: $G_{105} \setminus v_1$



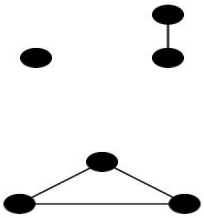
A.1.223: G_{106}



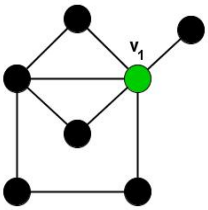
A.1.224: $G_{106} \setminus v_1$



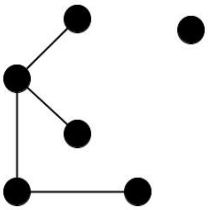
A.1.225: G_{107}



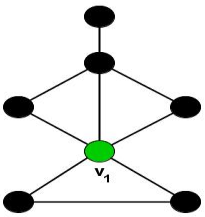
A.1.226: $G_{107} \setminus v_1$



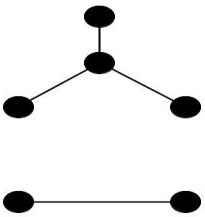
A.1.227: G_{108}



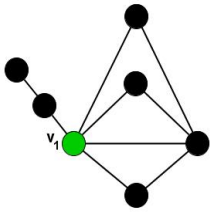
A.1.228: $G_{108} \setminus v_1$



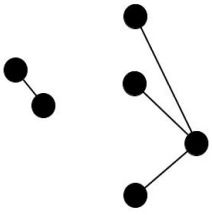
A.1.229: G_{109}



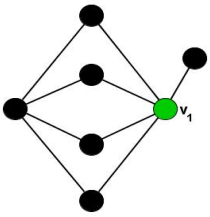
A.1.230: $G_{109} \setminus v_1$



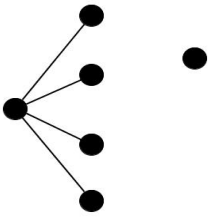
A.1.231: G_{110}



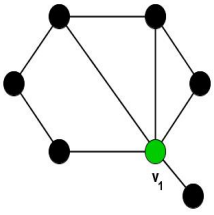
A.1.232: $G_{110} \setminus v_1$



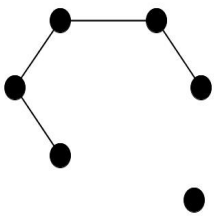
A.1.233: G_{111}



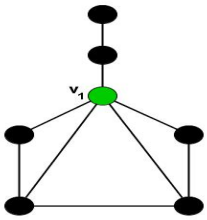
A.1.234: $G_{111} \setminus v_1$



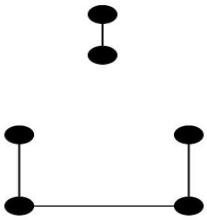
A.1.235: G_{112}



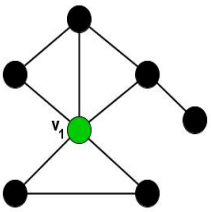
A.1.236: $G_{112} \setminus v_1$



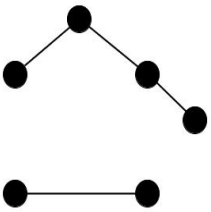
A.1.237: G_{113}



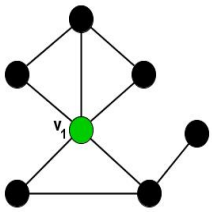
A.1.238: $G_{113} \setminus v_1$



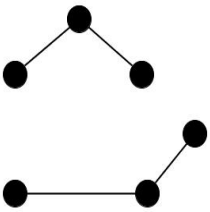
A.1.239: G_{114}



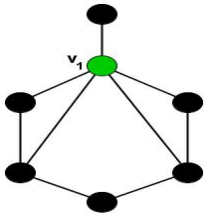
A.1.240: $G_{114} \setminus v_1$



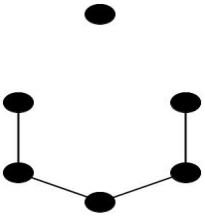
A.1.241: G_{115}



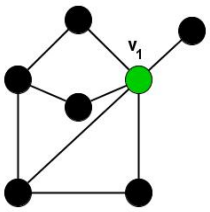
A.1.242: $G_{115} \setminus v_1$



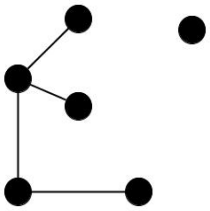
A.1.243: G_{116}



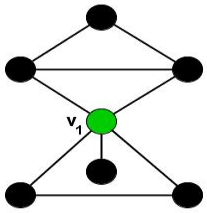
A.1.244: $G_{116} \setminus v_1$



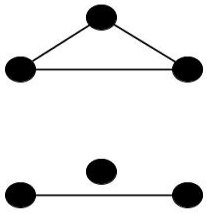
A.1.245: G_{117}



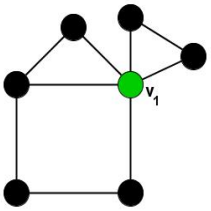
A.1.246: $G_{117} \setminus v_1$



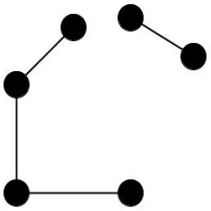
A.1.247: G_{118}



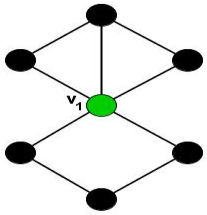
A.1.248: $G_{118} \setminus v_1$



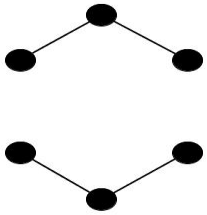
A.1.249: G_{119}



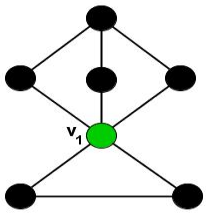
A.1.250: $G_{119} \setminus v_1$



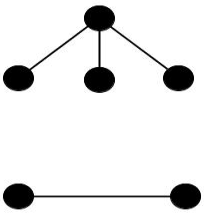
A.1.251: G_{120}



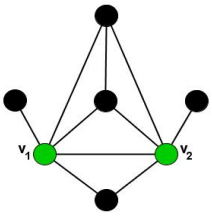
A.1.252: $G_{120} \setminus v_1$



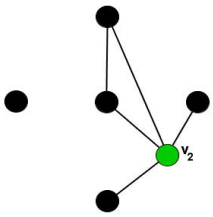
A.1.253: G_{121}



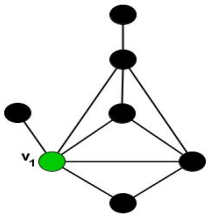
A.1.254: $G_{121} \setminus v_1$



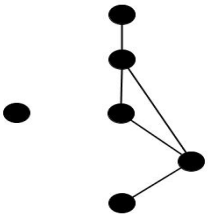
A.1.255: G_{122}



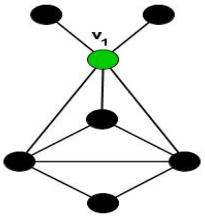
A.1.256: $G_{122} \setminus v_1$



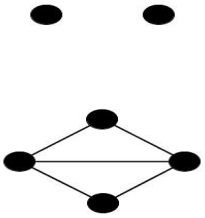
A.1.257: G_{123}



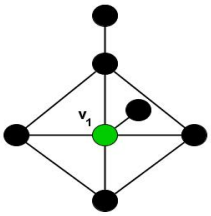
A.1.258: $G_{123} \setminus v_1$



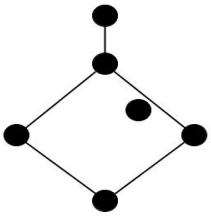
A.1.259: G_{124}



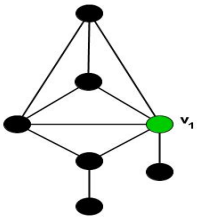
A.1.260: $G_{124} \setminus v_1$



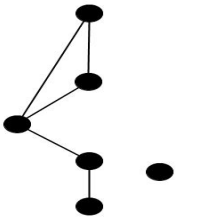
A.1.261: G_{125}



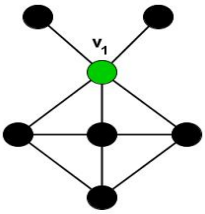
A.1.262: $G_{125} \setminus v_1$



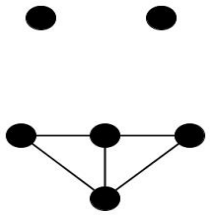
A.1.263: G_{126}



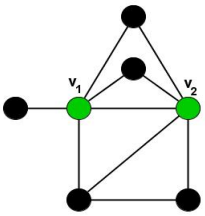
A.1.264: $G_{126} \setminus v_1$



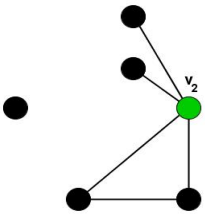
A.1.265: G_{127}



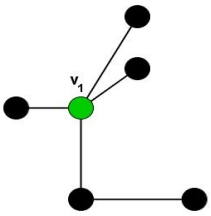
A.1.266: $G_{127} \setminus v_1$



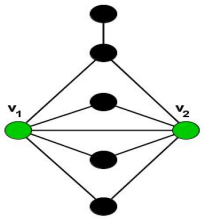
A.1.267: G_{128}



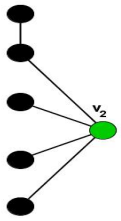
A.1.268: $G_{128} \setminus v_1$



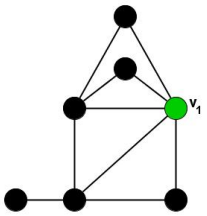
A.1.269: $G_{128} \setminus v_2$



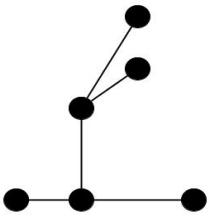
A.1.270: G_{129}



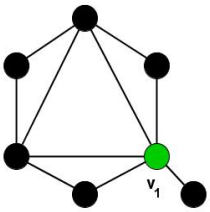
A.1.271: $G_{129} \setminus v_1$



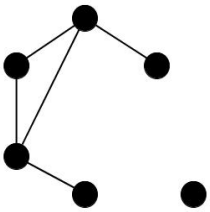
A.1.272: G_{130}



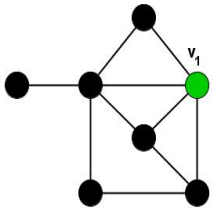
A.1.273: $G_{130} \setminus v_1$



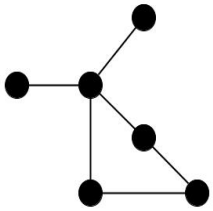
A.1.274: G_{131}



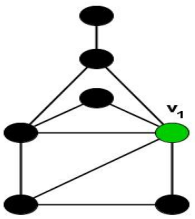
A.1.275: $G_{131} \setminus v_1$



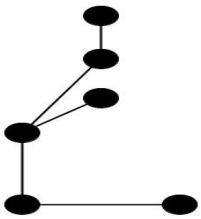
A.1.276: G_{132}



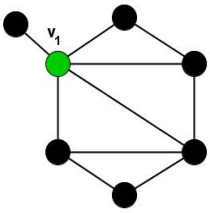
A.1.277: $G_{132} \setminus v_1$



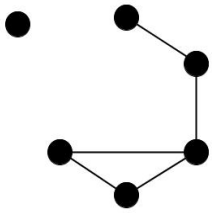
A.1.278: G_{133}



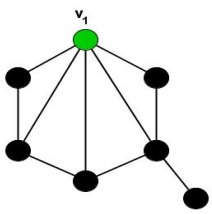
A.1.279: $G_{133} \setminus v_1$



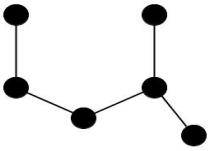
A.1.280: G_{134}



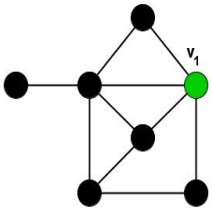
A.1.281: $G_{134} \setminus v_1$



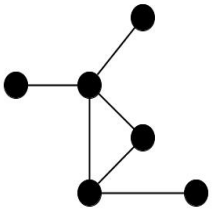
A.1.282: G_{135}



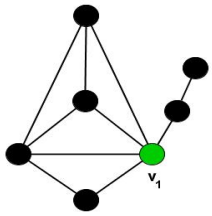
A.1.283: $G_{135} \setminus v_1$



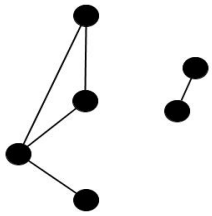
A.1.284: G_{136}



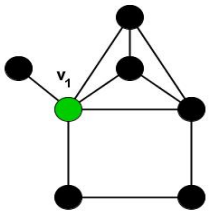
A.1.285: $G_{136} \setminus v_1$



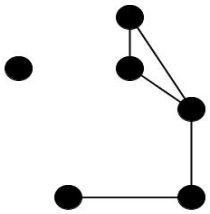
A.1.286: G_{137}



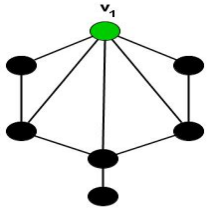
A.1.287: $G_{137} \setminus v_1$



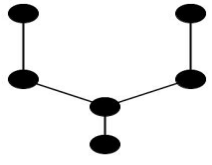
A.1.288: G_{138}



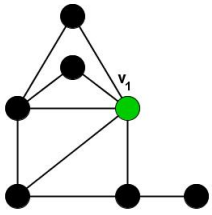
A.1.289: $G_{138} \setminus v_1$



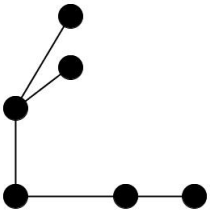
A.1.290: G_{139}



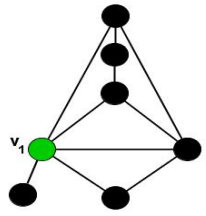
A.1.291: $G_{139} \setminus v_1$



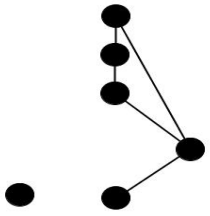
A.1.292: G_{140}



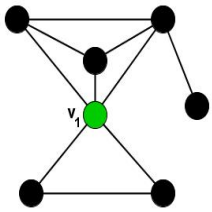
A.1.293: $G_{140} \setminus v_1$



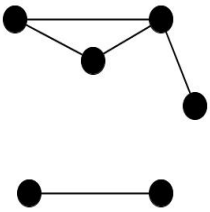
A.1.294: G_{141}



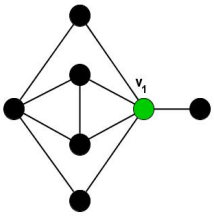
A.1.295: $G_{141} \setminus v_1$



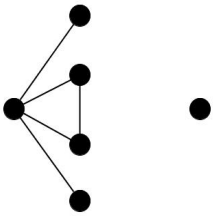
A.1.296: G_{142}



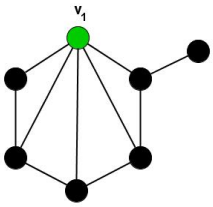
A.1.297: $G_{142} \setminus v_1$



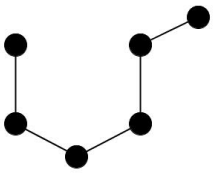
A.1.298: G_{143}



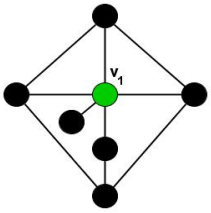
A.1.299: $G_{143} \setminus v_1$



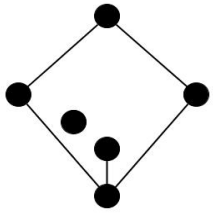
A.1.300: G_{144}



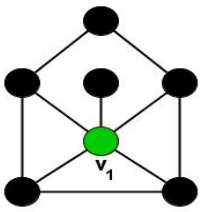
A.1.301: $G_{144} \setminus v_1$



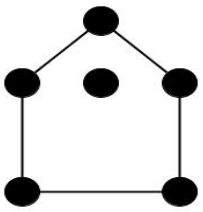
A.1.302: G_{145}



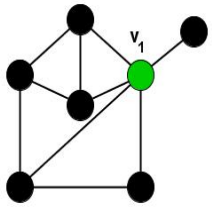
A.1.303: $G_{145} \setminus v_1$



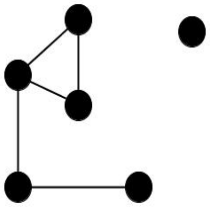
A.1.304: G_{146}



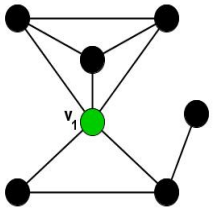
A.1.305: $G_{146} \setminus v_1$



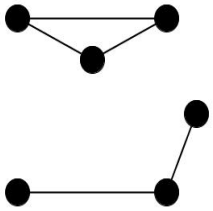
A.1.306: G_{147}



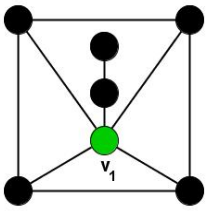
A.1.307: $G_{147} \setminus v_1$



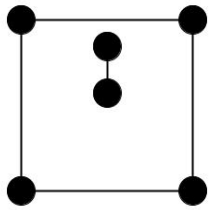
A.1.308: G_{148}



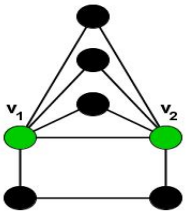
A.1.309: $G_{148} \setminus v_1$



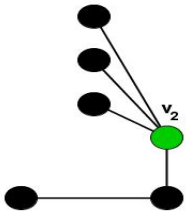
A.1.310: G_{149}



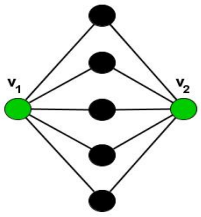
A.1.311: $G_{149} \setminus v_1$



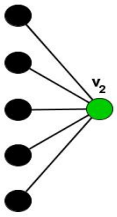
A.1.312: G_{150}



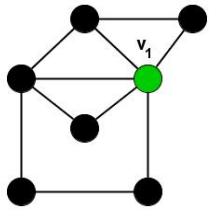
A.1.313: $G_{150} \setminus v_1$



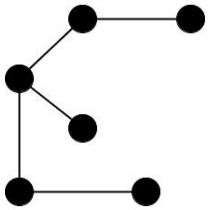
A.1.314: G_{151}



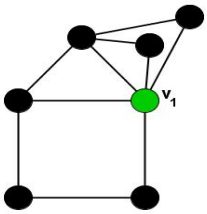
A.1.315: $G_{151} \setminus v_1$



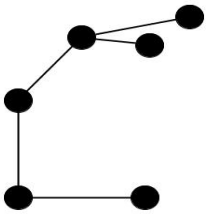
A.1.316: G_{152}



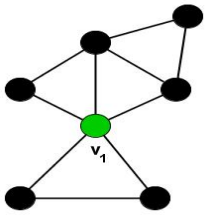
A.1.317: $G_{152} \setminus v_1$



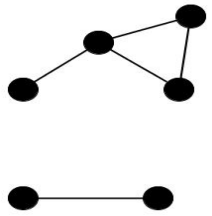
A.1.318: G_{153}



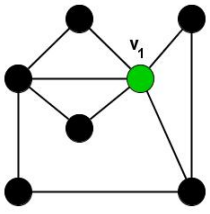
A.1.319: $G_{153} \setminus v_1$



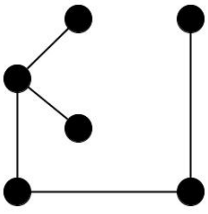
A.1.320: G_{154}



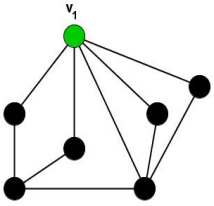
A.1.321: $G_{154} \setminus v_1$



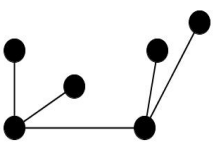
A.1.322: G_{155}



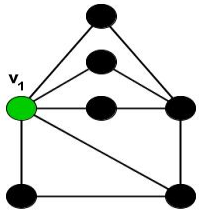
A.1.323: $G_{155} \setminus v_1$



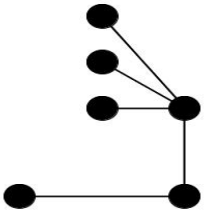
A.1.324: G_{156}



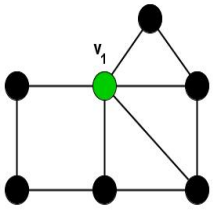
A.1.325: $G_{156} \setminus v_1$



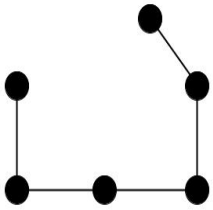
A.1.326: G_{157}



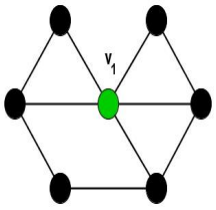
A.1.327: $G_{157} \setminus v_1$



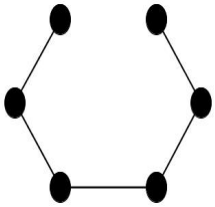
A.1.328: G_{158}



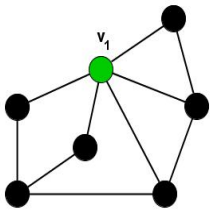
A.1.329: $G_{158} \setminus v_1$



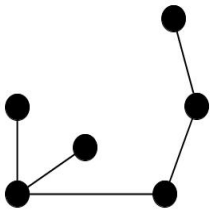
A.1.330: G_{159}



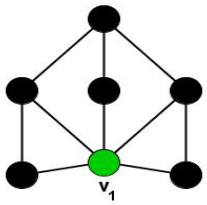
A.1.331: $G_{159} \setminus v_1$



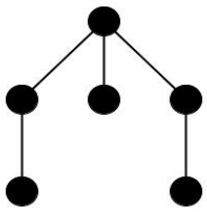
A.1.332: G_{160}



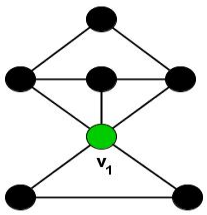
A.1.333: $G_{160} \setminus v_1$



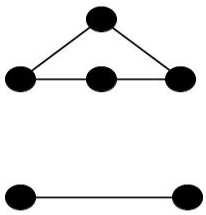
A.1.334: G_{161}



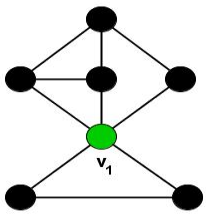
A.1.335: $G_{161} \setminus v_1$



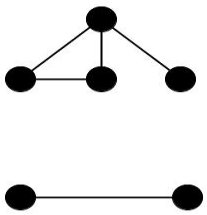
A.1.336: G_{162}



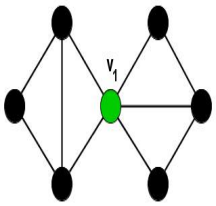
A.1.337: $G_{162} \setminus v_1$



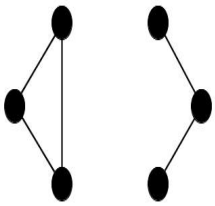
A.1.338: G_{163}



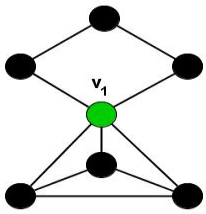
A.1.339: $G_{163} \setminus v_1$



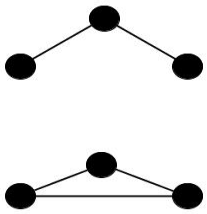
A.1.340: G_{164}



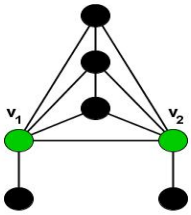
A.1.341: $G_{164} \setminus v_1$



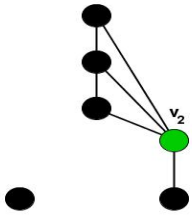
A.1.342: G_{165}



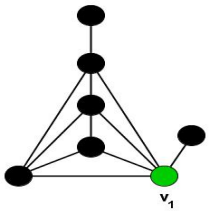
A.1.343: $G_{165} \setminus v_1$



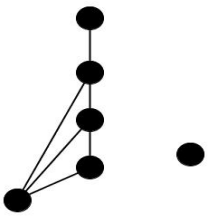
A.1.344: G_{166}



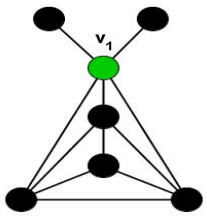
A.1.345: $G_{166} \setminus v_1$



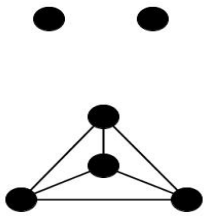
A.1.346: G_{167}



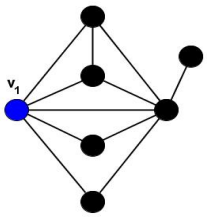
A.1.347: $G_{167} \setminus v_1$



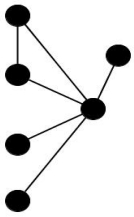
A.1.348: G_{168}



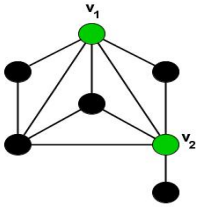
A.1.349: $G_{168} \setminus v_1$



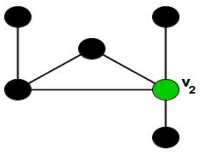
A.1.350: G_{169}



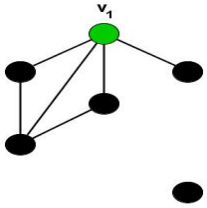
A.1.351: $G_{169} \setminus v_1$



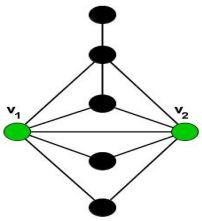
A.1.352: G_{170}



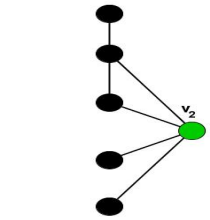
A.1.353: $G_{170} \setminus v_1$



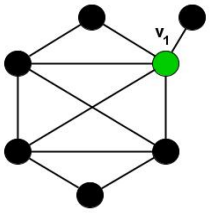
A.1.354: $G_{170} \setminus v_2$



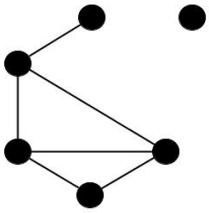
A.1.355: G_{171}



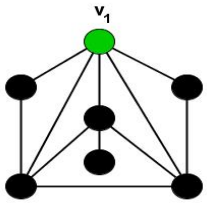
A.1.356: $G_{171} \setminus v_1$



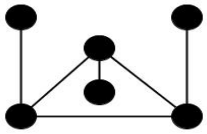
A.1.357: G_{172}



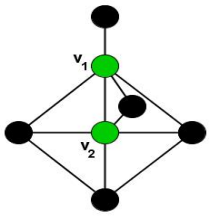
A.1.358: $G_{172} \setminus v_1$



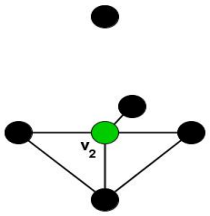
A.1.359: G_{173}



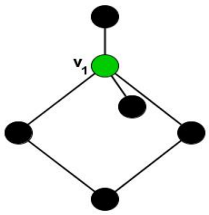
A.1.360: $G_{173} \setminus v_1$



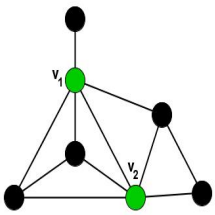
A.1.361: G_{174}



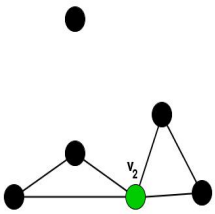
A.1.362: $G_{174} \setminus v_1$



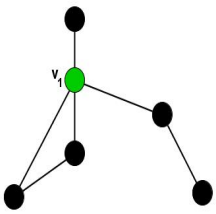
A.1.363: $G_{174} \setminus v_2$



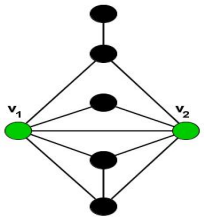
A.1.364: G_{175}



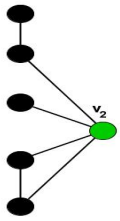
A.1.365: $G_{175} \setminus v_1$



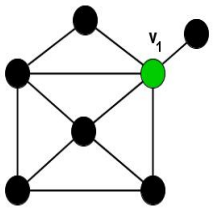
A.1.366: $G_{175} \setminus v_2$



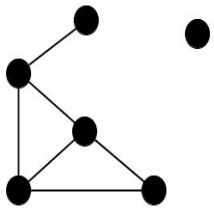
A.1.367: G_{176}



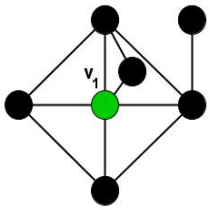
A.1.368: $G_{176} \setminus v_1$



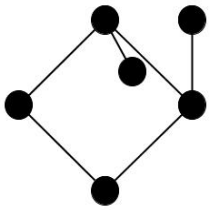
A.1.369: G_{177}



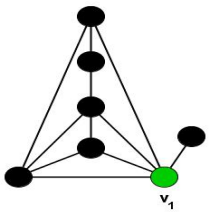
A.1.370: $G_{177} \setminus v_1$



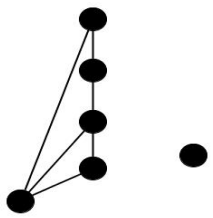
A.1.371: G_{178}



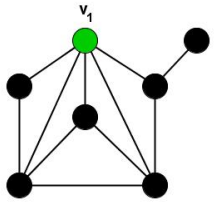
A.1.372: $G_{178} \setminus v_1$



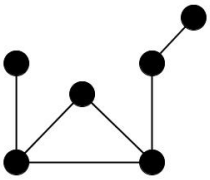
A.1.373: G_{179}



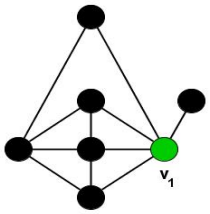
A.1.374: $G_{179} \setminus v_1$



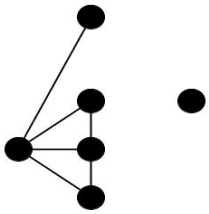
A.1.375: G_{180}



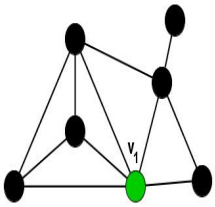
A.1.376: $G_{180} \setminus v_1$



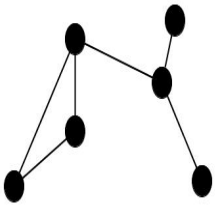
A.1.377: G_{181}



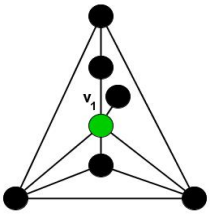
A.1.378: $G_{181} \setminus v_1$



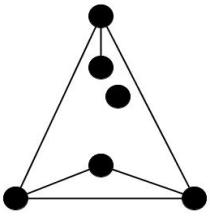
A.1.379: G_{182}



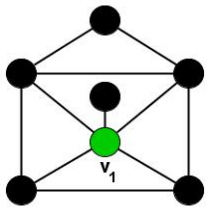
A.1.380: $G_{182} \setminus v_1$



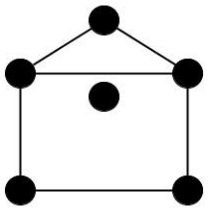
A.1.381: G_{183}



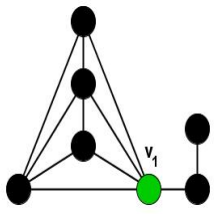
A.1.382: $G_{183} \setminus v_1$



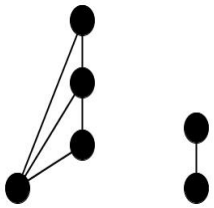
A.1.383: G_{184}



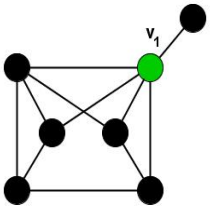
A.1.384: $G_{184} \setminus v_1$



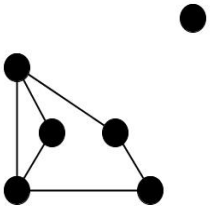
A.1.385: G_{185}



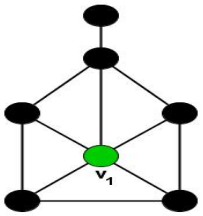
A.1.386: $G_{185} \setminus v_1$



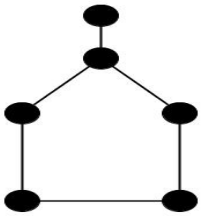
A.1.387: G_{186}



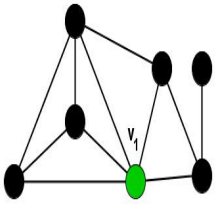
A.1.388: $G_{186} \setminus v_1$



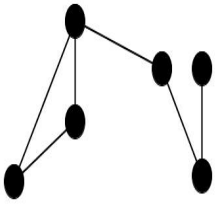
A.1.389: G_{187}



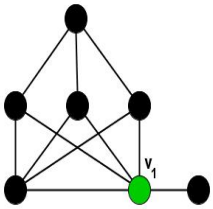
A.1.390: $G_{187} \setminus v_1$



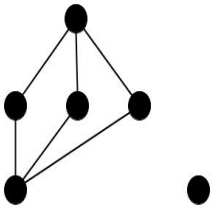
A.1.391: G_{188}



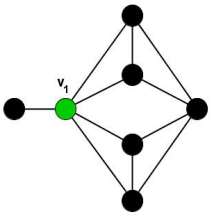
A.1.392: $G_{188} \setminus v_1$



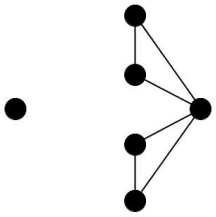
A.1.393: G_{189}



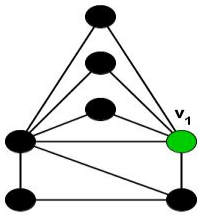
A.1.394: $G_{189} \setminus v_1$



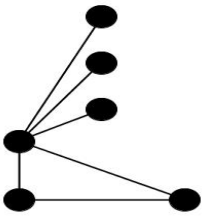
A.1.395: G_{190}



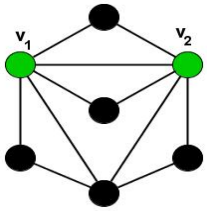
A.1.396: $G_{190} \setminus v_1$



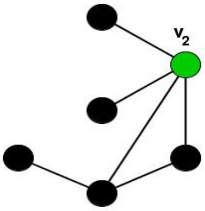
A.1.397: G_{191}



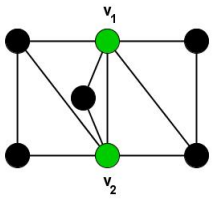
A.1.398: $G_{191} \setminus v_1$



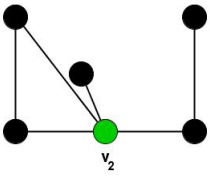
A.1.399: G_{192}



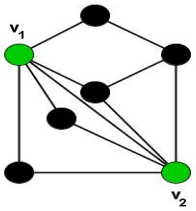
A.1.400: $G_{192} \setminus v_1$



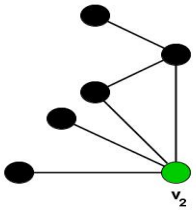
A.1.401: G_{193}



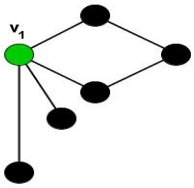
A.1.402: $G_{193} \setminus v_1$



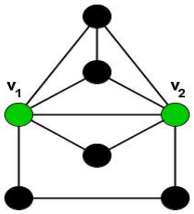
A.1.403: G_{194}



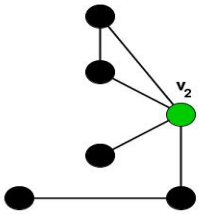
A.1.404: $G_{194} \setminus v_1$



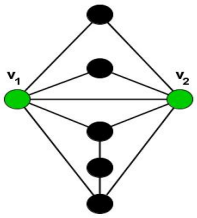
A.1.405: $G_{194} \setminus v_2$



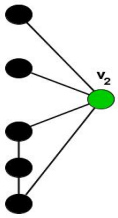
A.1.406: G_{195}



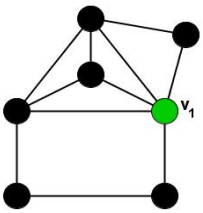
A.1.407: $G_{195} \setminus v_1$



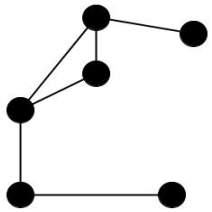
A.1.408: G_{196}



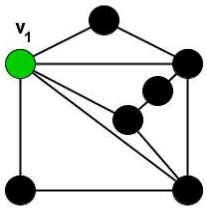
A.1.409: $G_{196} \setminus v_1$



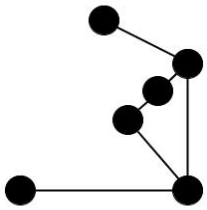
A.1.410: G_{197}



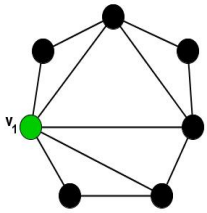
A.1.411: $G_{197} \setminus v_1$



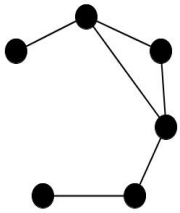
A.1.412: G_{198}



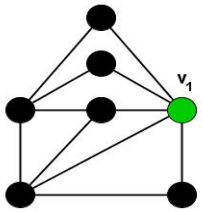
A.1.413: $G_{198} \setminus v_1$



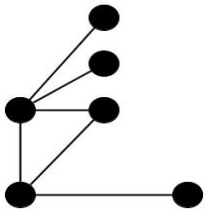
A.1.414: G_{199}



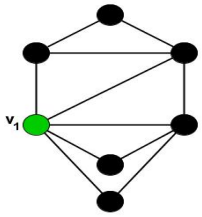
A.1.415: $G_{199} \setminus v_1$



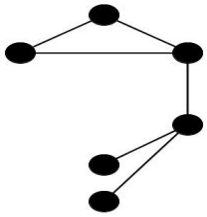
A.1.416: G_{200}



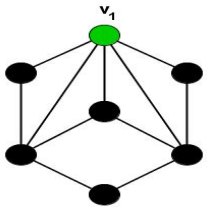
A.1.417: $G_{200} \setminus v_1$



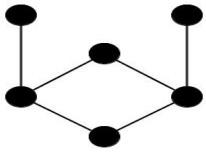
A.1.418: G_{201}



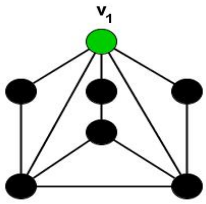
A.1.419: $G_{201} \setminus v_1$



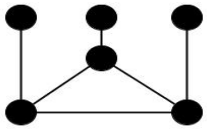
A.1.420: G_{202}



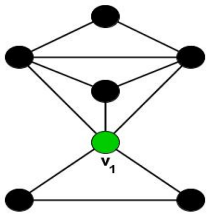
A.1.421: $G_{202} \setminus v_1$



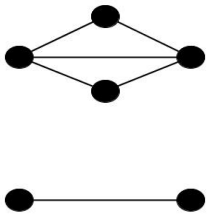
A.1.422: G_{203}



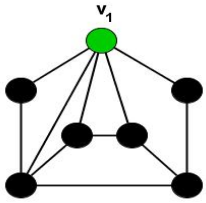
A.1.423: $G_{203} \setminus v_1$



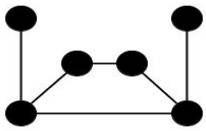
A.1.424: G_{204}



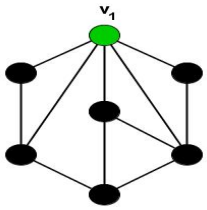
A.1.425: $G_{204} \setminus v_1$



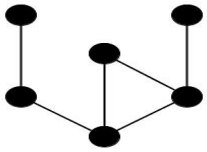
A.1.426: G_{205}



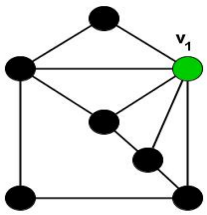
A.1.427: $G_{205} \setminus v_1$



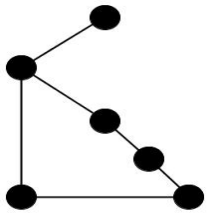
A.1.428: G_{206}



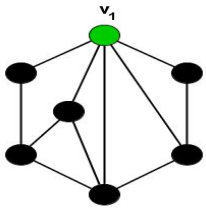
A.1.429: $G_{206} \setminus v_1$



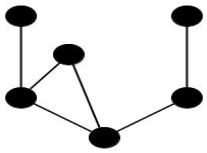
A.1.430: G_{207}



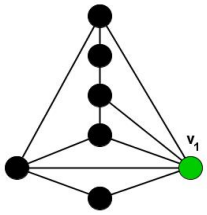
A.1.431: $G_{207} \setminus v_1$



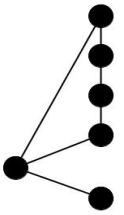
A.1.432: G_{208}



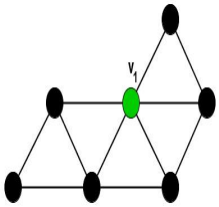
A.1.433: $G_{208} \setminus v_1$



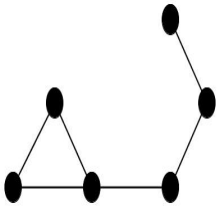
A.1.434: G_{209}



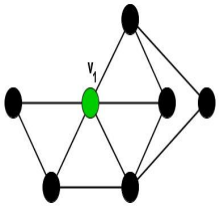
A.1.435: $G_{209} \setminus v_1$



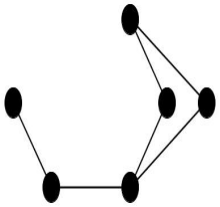
A.1.436: G_{210}



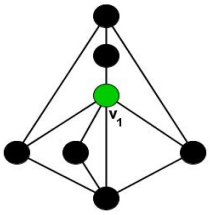
A.1.437: $G_{210} \setminus v_1$



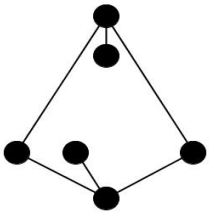
A.1.438: G_{211}



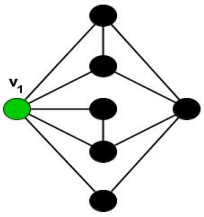
A.1.439: $G_{211} \setminus v_1$



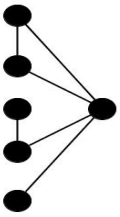
A.1.440: G_{212}



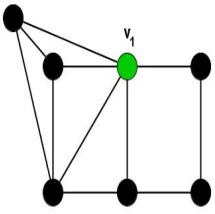
A.1.441: $G_{212} \setminus v_1$



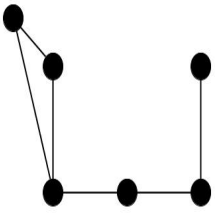
A.1.442: G_{213}



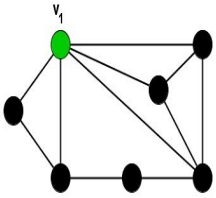
A.1.443: $G_{213} \setminus v_1$



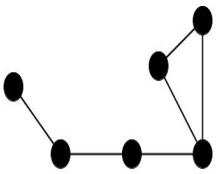
A.1.444: G_{214}



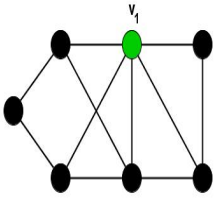
A.1.445: $G_{214} \setminus v_1$



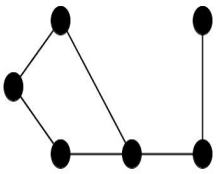
A.1.446: G_{215}



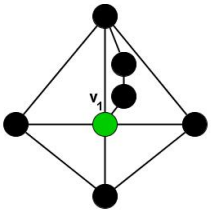
A.1.447: $G_{215} \setminus v_1$



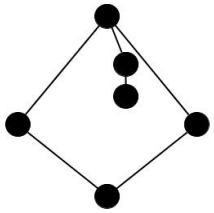
A.1.448: G_{216}



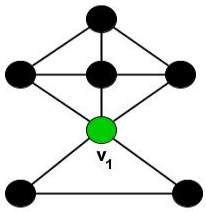
A.1.449: $G_{216} \setminus v_1$



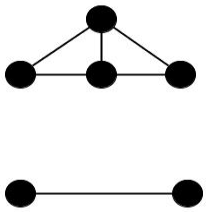
A.1.450: G_{217}



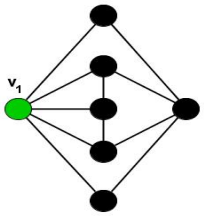
A.1.451: $G_{217} \setminus v_1$



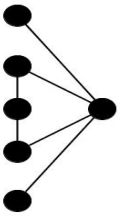
A.1.452: G_{218}



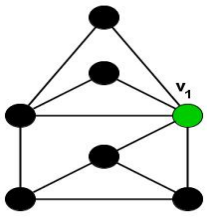
A.1.453: $G_{218} \setminus v_1$



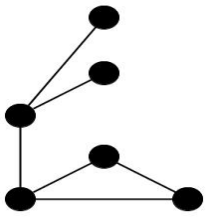
A.1.454: G_{219}



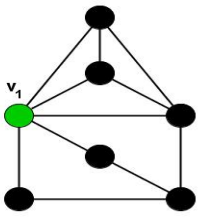
A.1.455: $G_{219} \setminus v_1$



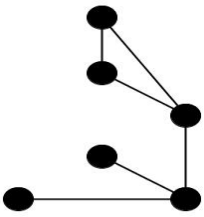
A.1.456: G_{220}



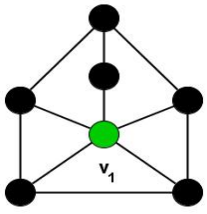
A.1.457: $G_{220} \setminus v_1$



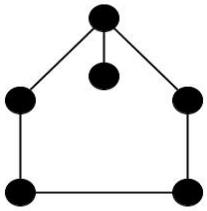
A.1.458: G_{221}



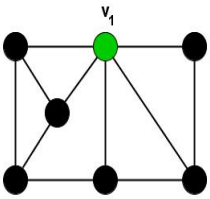
A.1.459: $G_{221} \setminus v_1$



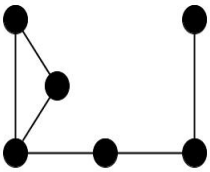
A.1.460: G_{222}



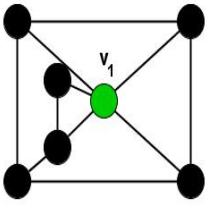
A.1.461: $G_{222} \setminus v_1$



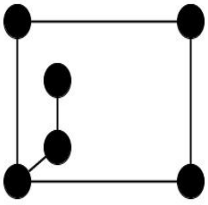
A.1.462: G_{223}



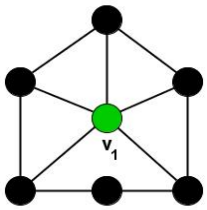
A.1.463: $G_{223} \setminus v_1$



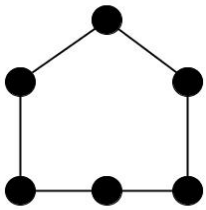
A.1.464: G_{224}



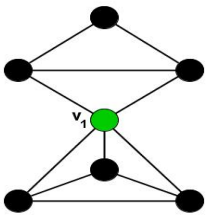
A.1.465: $G_{224} \setminus v_1$



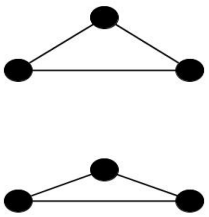
A.1.466: G_{225}



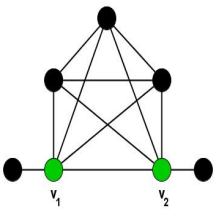
A.1.467: $G_{225} \setminus v_1$



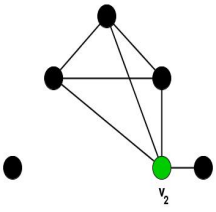
A.1.468: G_{226}



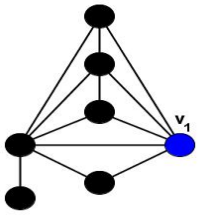
A.1.469: $G_{226} \setminus v_1$



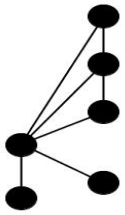
A.1.470: G_{227}



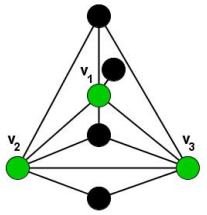
A.1.471: $G_{227} \setminus v_1$



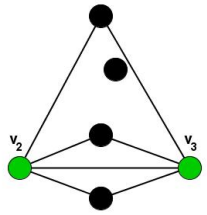
A.1.472: G_{228}



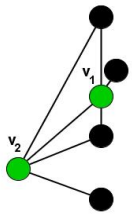
A.1.473: $G_{228} \setminus v_1$



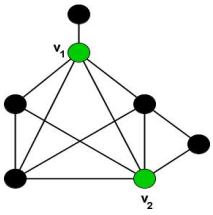
A.1.474: G_{229}



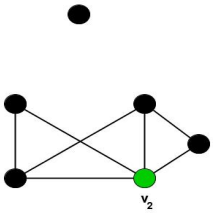
A.1.475: $G_{229} \setminus v_1$



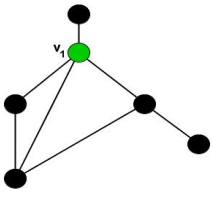
A.1.476: $G_{229} \setminus v_3$



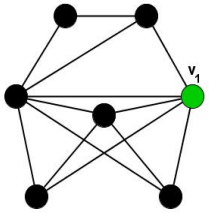
A.1.477: G_{230}



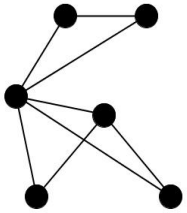
A.1.478: $G_{230} \setminus v_1$



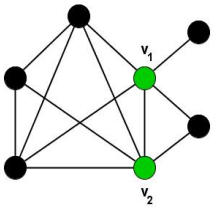
A.1.479: $G_{230} \setminus v_2$



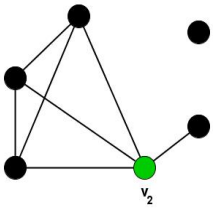
A.1.480: G_{231}



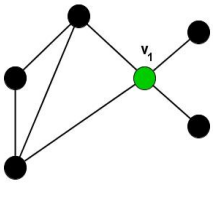
A.1.481: $G_{231} \setminus v_1$



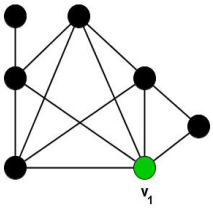
A.1.482: G_{232}



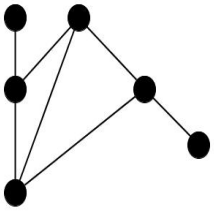
A.1.483: $G_{232} \setminus v_1$



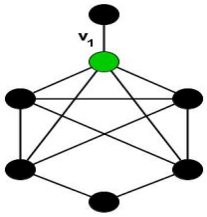
A.1.484: $G_{232} \setminus v_2$



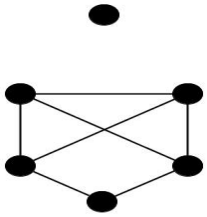
A.1.485: G_{233}



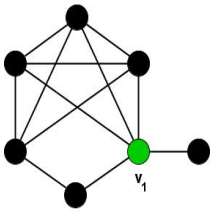
A.1.486: $G_{233} \setminus v_1$



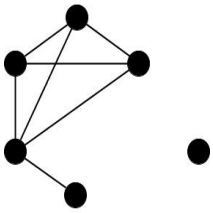
A.1.487: G_{234}



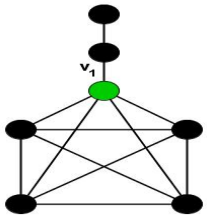
A.1.488: $G_{234} \setminus v_1$



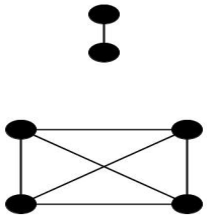
A.1.489: G_{235}



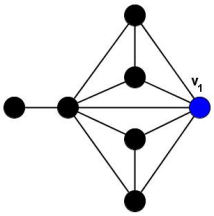
A.1.490: $G_{235} \setminus v_1$



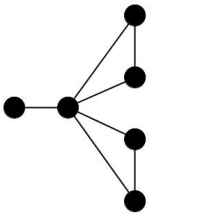
A.1.491: G_{236}



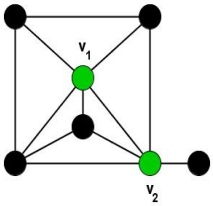
A.1.492: $G_{236} \setminus v_1$



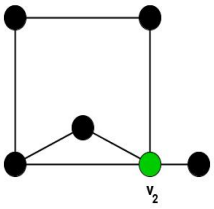
A.1.493: G_{237}



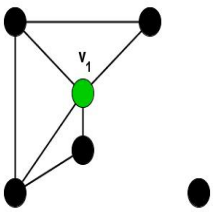
A.1.494: $G_{237} \setminus v_1$



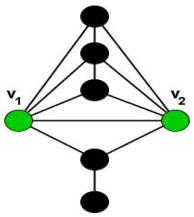
A.1.495: G_{238}



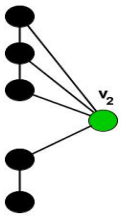
A.1.496: $G_{238} \setminus v_1$



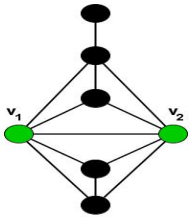
A.1.497: $G_{238} \setminus v_2$



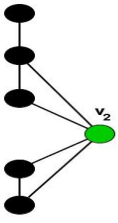
A.1.498: G_{239}



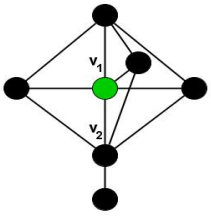
A.1.499: $G_{239} \setminus v_1$



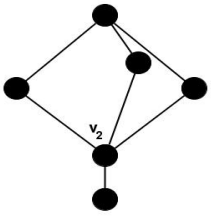
A.1.500: G_{240}



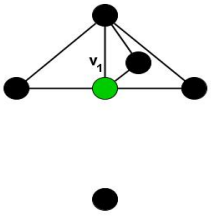
A.1.501: $G_{240} \setminus v_1$



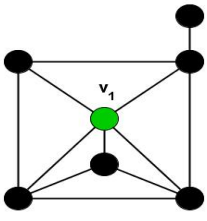
A.1.502: G_{241}



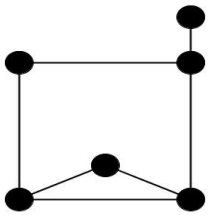
A.1.503: $G_{241} \setminus v_1$



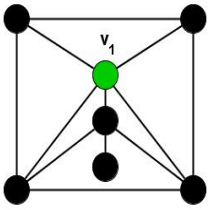
A.1.504: $G_{241} \setminus v_2$



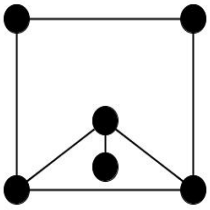
A.1.505: G_{242}



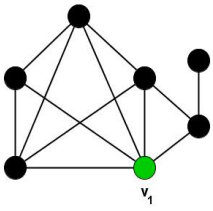
A.1.506: $G_{242} \setminus v_1$



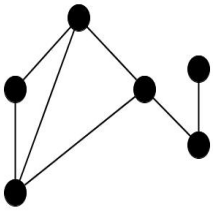
A.1.507: G_{243}



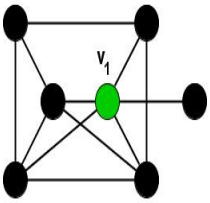
A.1.508: $G_{243} \setminus v_1$



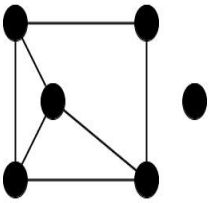
A.1.509: G_{244}



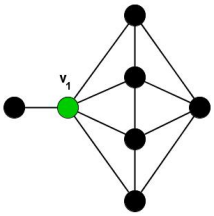
A.1.510: $G_{244} \setminus v_1$



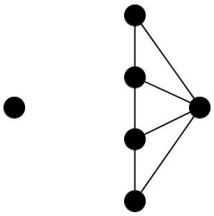
A.1.511: G_{245}



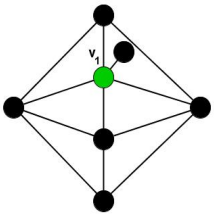
A.1.512: $G_{245} \setminus v_1$



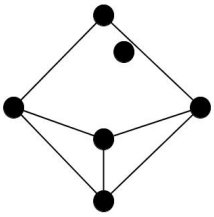
A.1.513: G_{246}



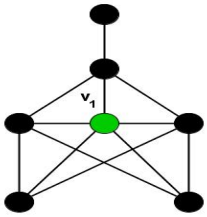
A.1.514: $G_{246} \setminus v_1$



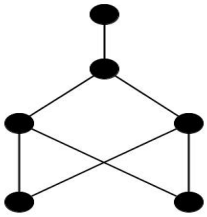
A.1.515: G_{247}



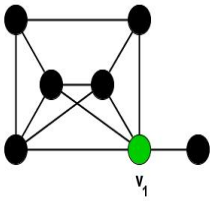
A.1.516: $G_{247} \setminus v_1$



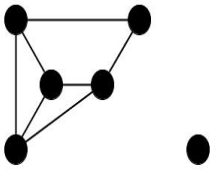
A.1.517: G_{248}



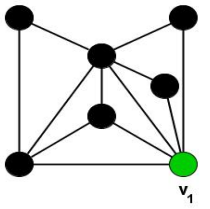
A.1.518: $G_{248} \setminus v_1$



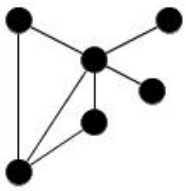
A.1.519: G_{249}



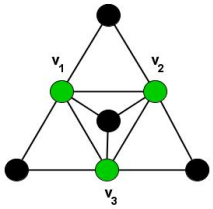
A.1.520: $G_{249} \setminus v_1$



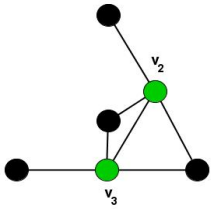
A.1.521: G_{250}



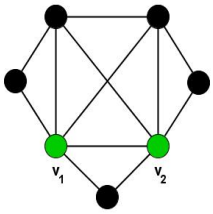
A.1.522: $G_{250} \setminus v_1$



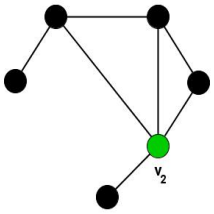
A.1.523: G_{251}



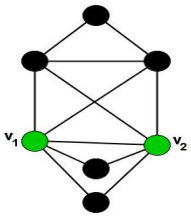
A.1.524: $G_{251} \setminus v_1$



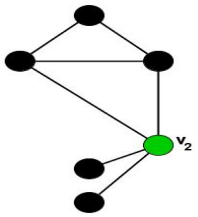
A.1.525: G_{252}



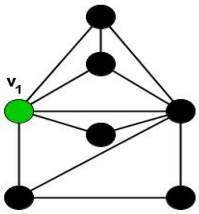
A.1.526: $G_{252} \setminus v_1$



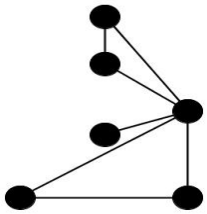
A.1.527: G_{253}



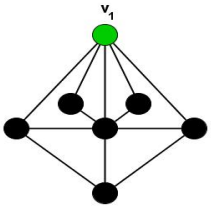
A.1.528: $G_{253} \setminus v_1$



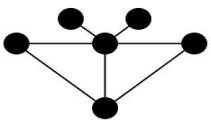
A.1.529: G_{254}



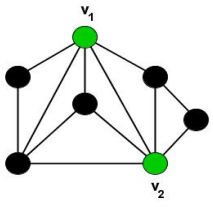
A.1.530: $G_{254} \setminus v_1$



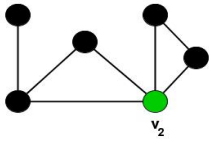
A.1.531: G_{255}



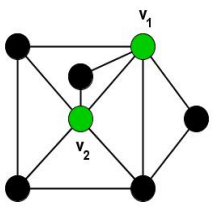
A.1.532: $G_{255} \setminus v_1$



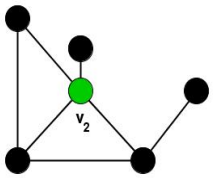
A.1.533: G_{256}



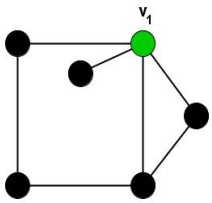
A.1.534: $G_{256} \setminus v_1$



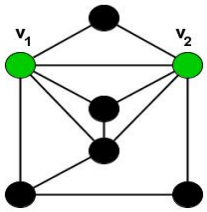
A.1.535: G_{257}



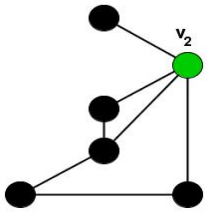
A.1.536: $G_{257} \setminus v_1$



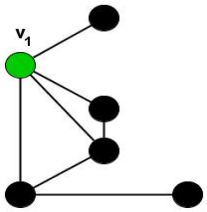
A.1.537: $G_{257} \setminus v_2$



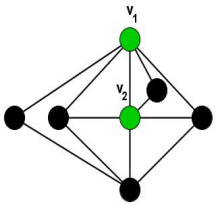
A.1.538: G_{258}



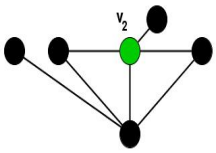
A.1.539: $G_{258} \setminus v_1$



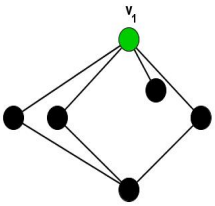
A.1.540: $G_{258} \setminus v_2$



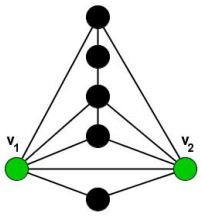
A.1.541: G_{259}



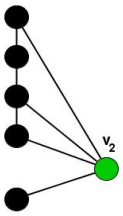
A.1.542: $G_{259} \setminus v_1$



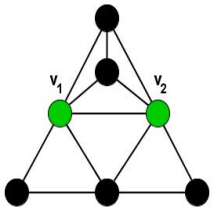
A.1.543: $G_{259} \setminus v_2$



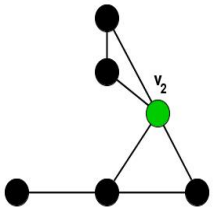
A.1.544: G_{260}



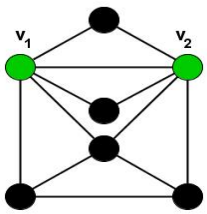
A.1.545: $G_{260} \setminus v_1$



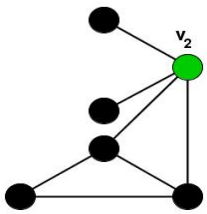
A.1.546: G_{261}



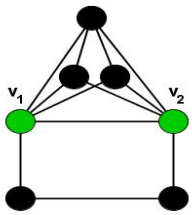
A.1.547: $G_{261} \setminus v_1$



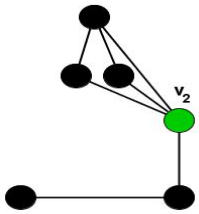
A.1.548: G_{262}



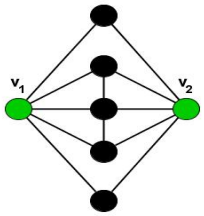
A.1.549: $G_{262} \setminus v_1$



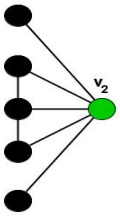
A.1.550: G_{263}



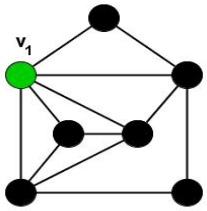
A.1.551: $G_{263} \setminus v_1$



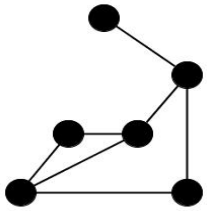
A.1.552: G_{264}



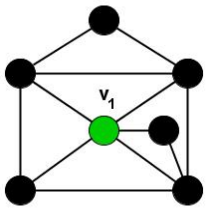
A.1.553: $G_{264} \setminus v_1$



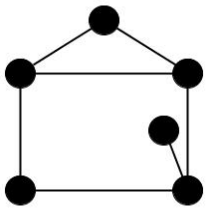
A.1.554: G_{265}



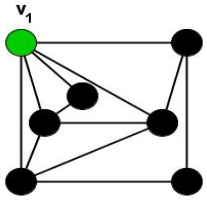
A.1.555: $G_{265} \setminus v_1$



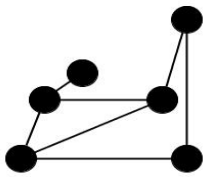
A.1.556: G_{266}



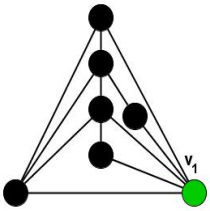
A.1.557: $G_{266} \setminus v_1$



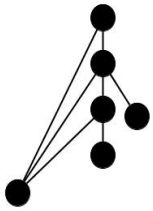
A.1.558: G_{267}



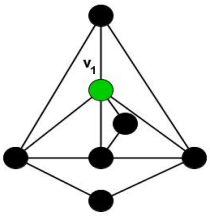
A.1.559: $G_{267} \setminus v_1$



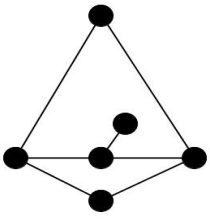
A.1.560: G_{268}



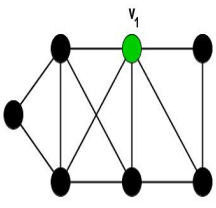
A.1.561: $G_{268} \setminus v_1$



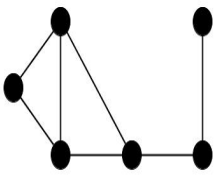
A.1.562: G_{269}



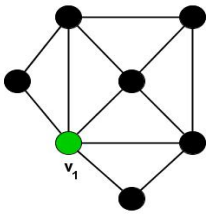
A.1.563: $G_{269} \setminus v_1$



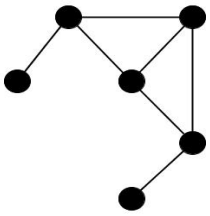
A.1.564: G_{270}



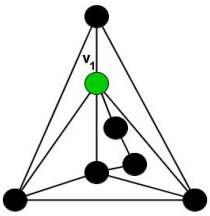
A.1.565: $G_{270} \setminus v_1$



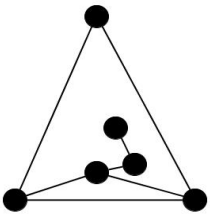
A.1.566: G_{271}



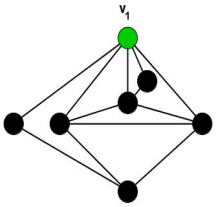
A.1.567: $G_{271} \setminus v_1$



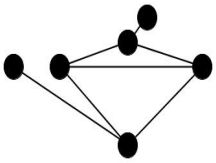
A.1.568: G_{272}



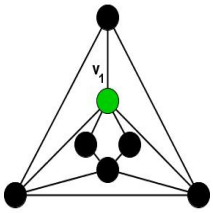
A.1.569: $G_{272} \setminus v_1$



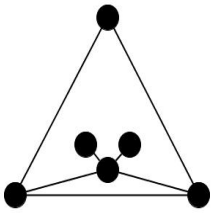
A.1.570: G_{273}



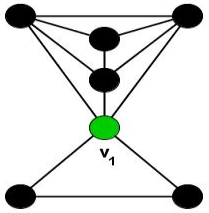
A.1.571: $G_{273} \setminus v_1$



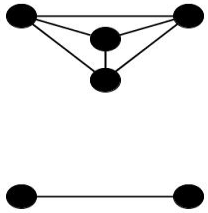
A.1.572: G_{274}



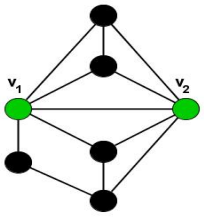
A.1.573: $G_{274} \setminus v_1$



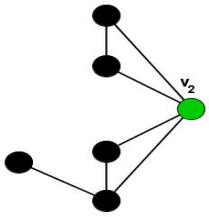
A.1.574: G_{275}



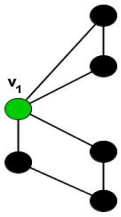
A.1.575: $G_{275} \setminus v_1$



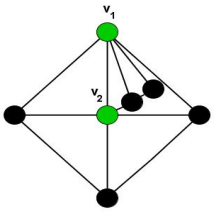
A.1.576: G_{276}



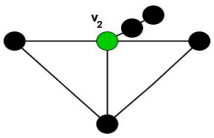
A.1.577: $G_{276} \setminus v_1$



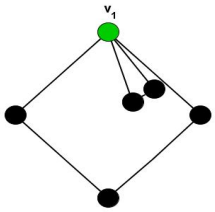
A.1.578: $G_{276} \setminus v_2$



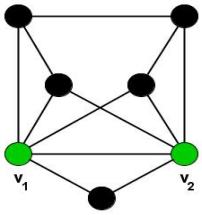
A.1.579: G_{277}



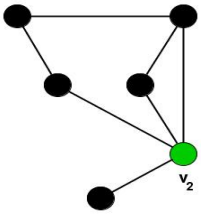
A.1.580: $G_{277} \setminus v_1$



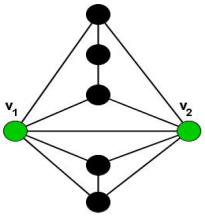
A.1.581: $G_{277} \setminus v_2$



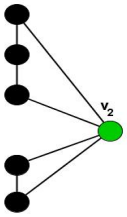
A.1.582: G_{278}



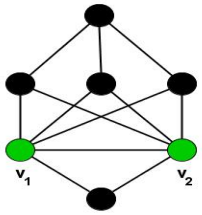
A.1.583: $G_{278} \setminus v_1$



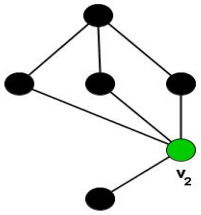
A.1.584: G_{279}



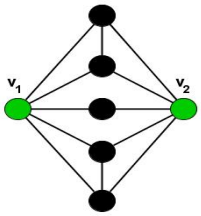
A.1.585: $G_{279} \setminus v_1$



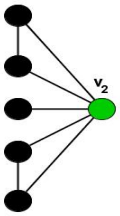
A.1.586: G_{280}



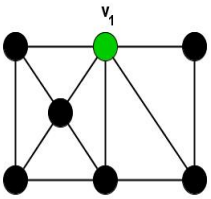
A.1.587: $G_{280} \setminus v_1$



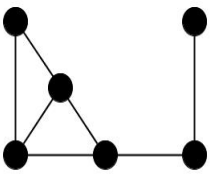
A.1.588: G_{281}



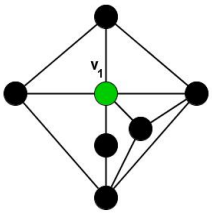
A.1.589: $G_{281} \setminus v_1$



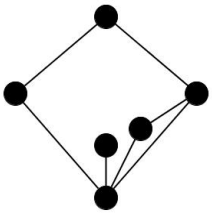
A.1.590: G_{282}



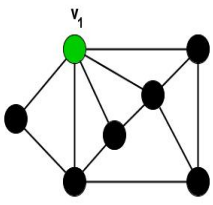
A.1.591: $G_{282} \setminus v_1$



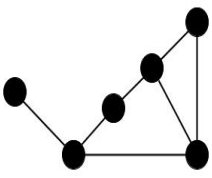
A.1.592: G_{283}



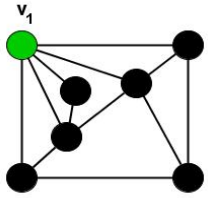
A.1.593: $G_{283} \setminus v_1$



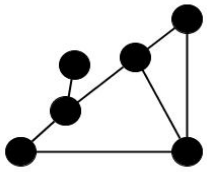
A.1.594: G_{284}



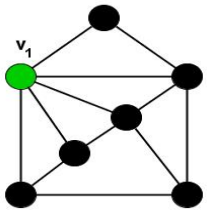
A.1.595: $G_{284} \setminus v_1$



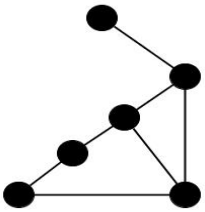
A.1.596: G_{285}



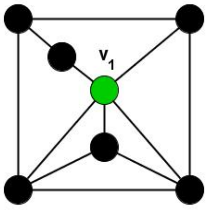
A.1.597: $G_{285} \setminus v_1$



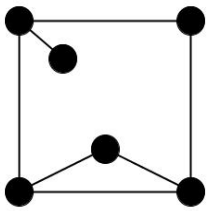
A.1.598: G_{286}



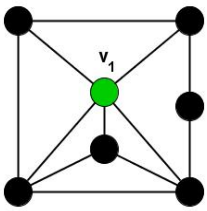
A.1.599: $G_{286} \setminus v_1$



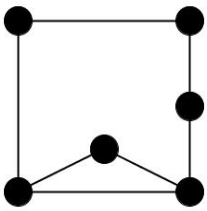
A.1.600: G_{287}



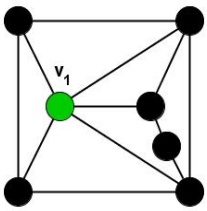
A.1.601: $G_{287} \setminus v_1$



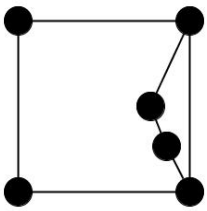
A.1.602: G_{288}



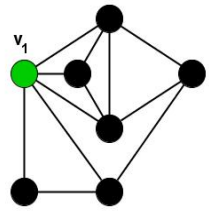
A.1.603: $G_{288} \setminus v_1$



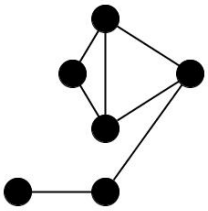
A.1.604: G_{289}



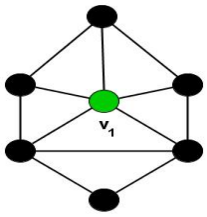
A.1.605: $G_{289} \setminus v_1$



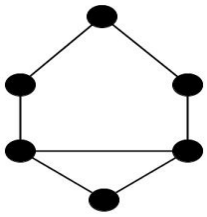
A.1.606: G_{290}



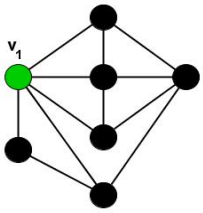
A.1.607: $G_{290} \setminus v_1$



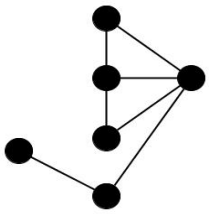
A.1.608: G_{291}



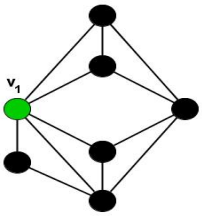
A.1.609: $G_{291} \setminus v_1$



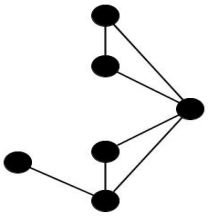
A.1.610: G_{292}



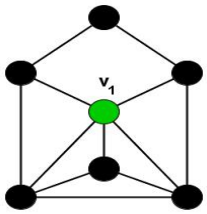
A.1.611: $G_{292} \setminus v_1$



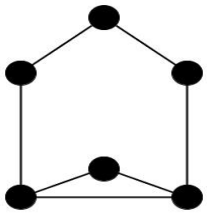
A.1.612: G_{293}



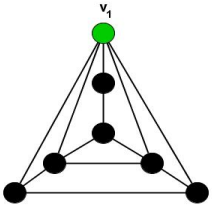
A.1.613: $G_{293} \setminus v_1$



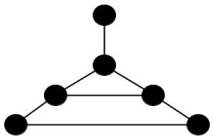
A.1.614: G_{294}



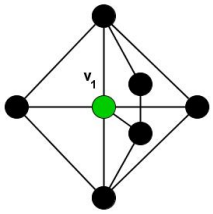
A.1.615: $G_{294} \setminus v_1$



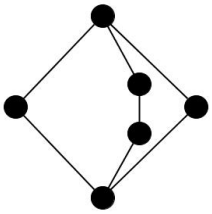
A.1.616: G_{295}



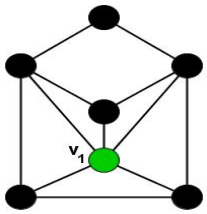
A.1.617: $G_{295} \setminus v_1$



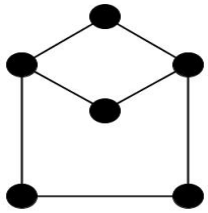
A.1.618: G_{296}



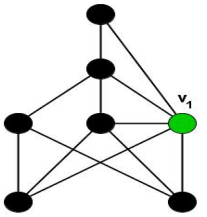
A.1.619: $G_{296} \setminus v_1$



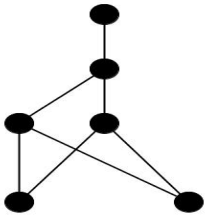
A.1.620: G_{297}



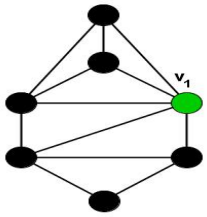
A.1.621: $G_{297} \setminus v_1$



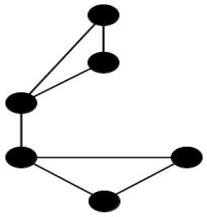
A.1.622: G_{298}



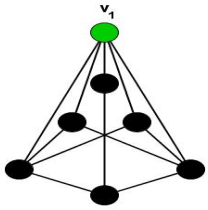
A.1.623: $G_{298} \setminus v_1$



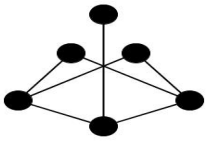
A.1.624: G_{299}



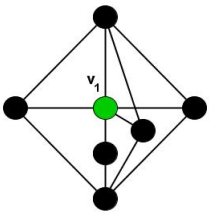
A.1.625: $G_{299} \setminus v_1$



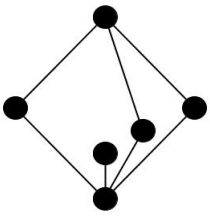
A.1.626: G_{300}



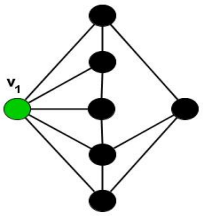
A.1.627: $G_{300} \setminus v_1$



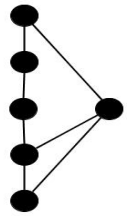
A.1.628: G_{301}



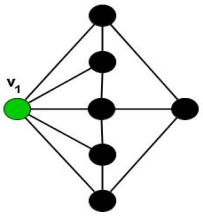
A.1.629: $G_{301} \setminus v_1$



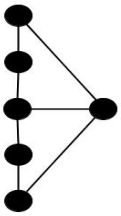
A.1.630: G_{302}



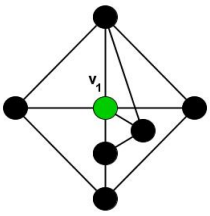
A.1.631: $G_{302} \setminus v_1$



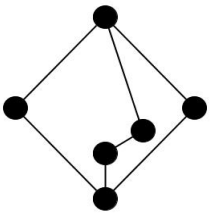
A.1.632: G_{303}



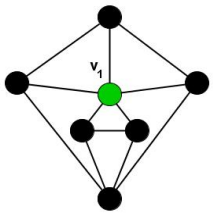
A.1.633: $G_{303} \setminus v_1$



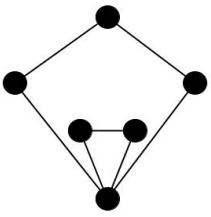
A.1.634: G_{304}



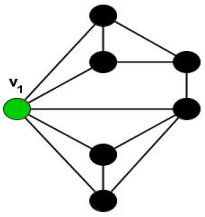
A.1.635: $G_{304} \setminus v_1$



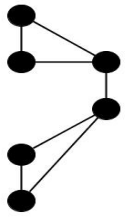
A.1.636: G_{305}



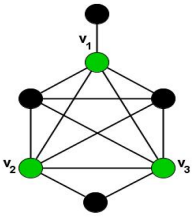
A.1.637: $G_{305} \setminus v_1$



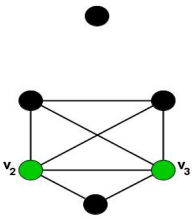
A.1.638: G_{306}



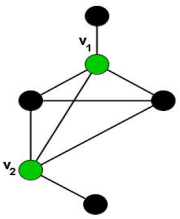
A.1.639: $G_{306} \setminus v_1$



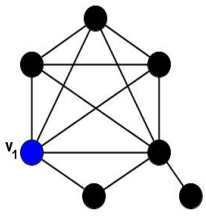
A.1.640: G_{307}



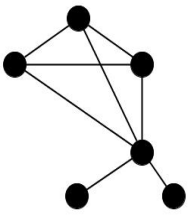
A.1.641: $G_{307} \setminus v_1$



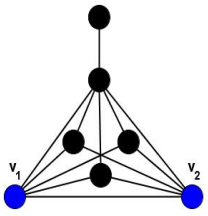
A.1.642: $G_{307} \setminus v_3$



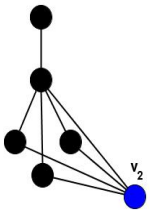
A.1.643: G_{308}



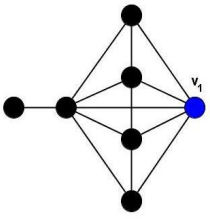
A.1.644: $G_{308} \setminus v_1$



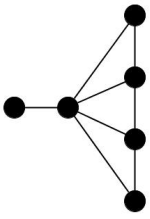
A.1.645: G_{309}



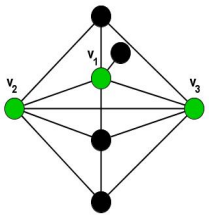
A.1.646: $G_{309} \setminus v_1$



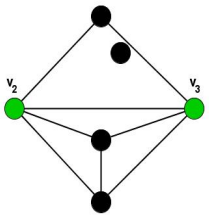
A.1.647: G_{310}



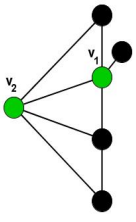
A.1.648: $G_{310} \setminus v_1$



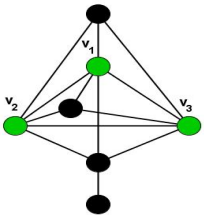
A.1.649: G_{311}



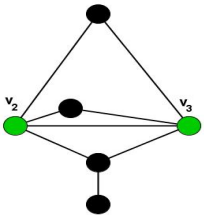
A.1.650: $G_{311} \setminus v_1$



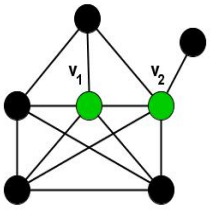
A.1.651: $G_{311} \setminus v_3$



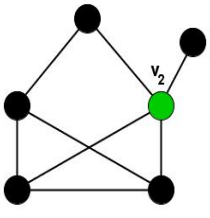
A.1.652: G_{312}



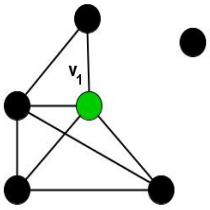
A.1.653: $G_{312} \setminus v_1$



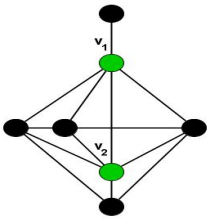
A.1.654: G_{313}



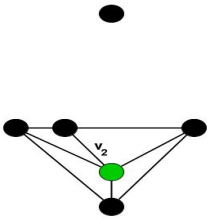
A.1.655: $G_{313} \setminus v_1$



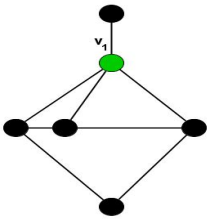
A.1.656: $G_{313} \setminus v_2$



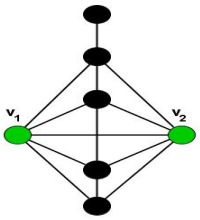
A.1.657: G_{314}



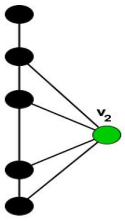
A.1.658: $G_{314} \setminus v_1$



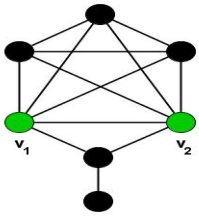
A.1.659: $G_{314} \setminus v_2$



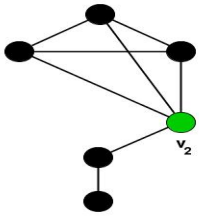
A.1.660: G_{315}



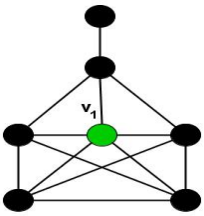
A.1.661: $G_{315} \setminus v_1$



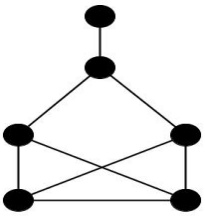
A.1.662: G_{316}



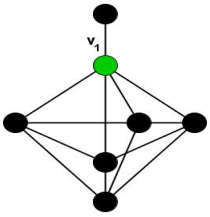
A.1.663: $G_{316} \setminus v_1$



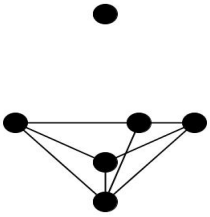
A.1.664: G_{317}



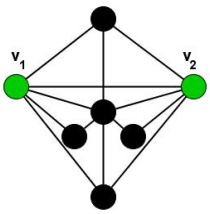
A.1.665: $G_{317} \setminus v_1$



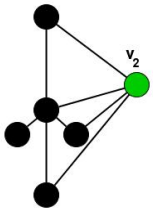
A.1.666: G_{318}



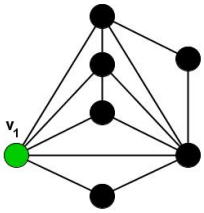
A.1.667: $G_{318} \setminus v_1$



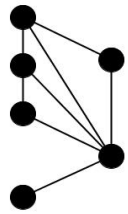
A.1.668: G_{319}



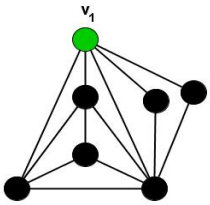
A.1.669: $G_{319} \setminus v_1$



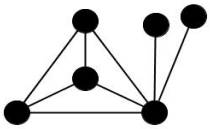
A.1.670: G_{320}



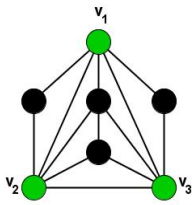
A.1.671: $G_{320} \setminus v_1$



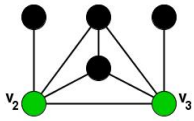
A.1.672: G_{321}



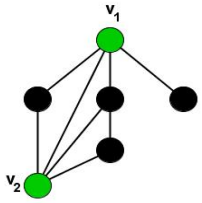
A.1.673: $G_{321} \setminus v_1$



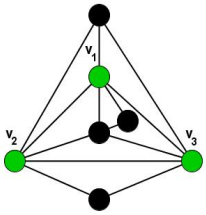
A.1.674: G_{322}



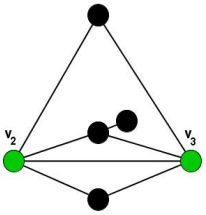
A.1.675: $G_{322} \setminus v_1$



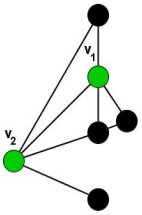
A.1.676: $G_{322} \setminus v_3$



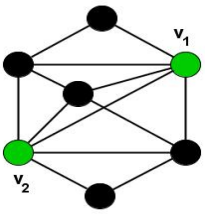
A.1.677: G_{323}



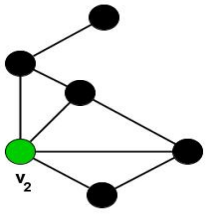
A.1.678: $G_{323} \setminus v_1$



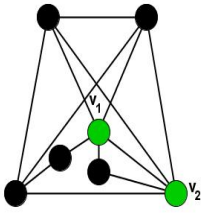
A.1.679: $G_{323} \setminus v_3$



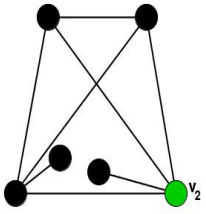
A.1.680: G_{324}



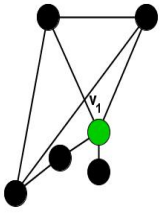
A.1.681: $G_{324} \setminus v_1$



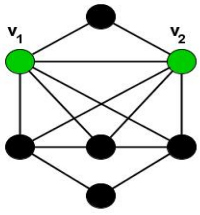
A.1.682: G_{325}



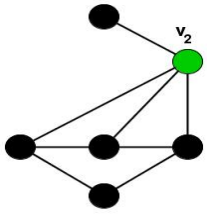
A.1.683: $G_{325} \setminus v_1$



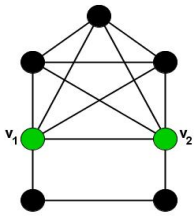
A.1.684: $G_{325} \setminus v_2$



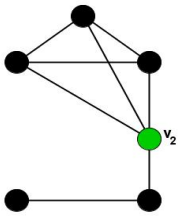
A.1.685: G_{326}



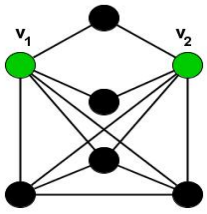
A.1.686: $G_{326} \setminus v_1$



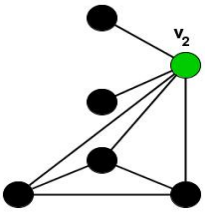
A.1.687: G_{327}



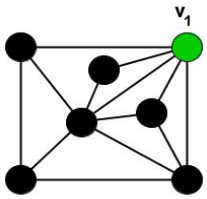
A.1.688: $G_{327} \setminus v_1$



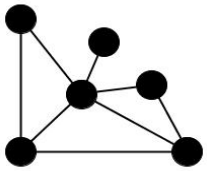
A.1.689: G_{328}



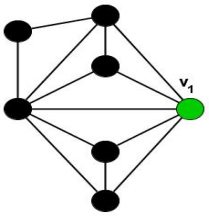
A.1.690: $G_{328} \setminus v_1$



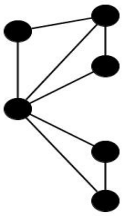
A.1.691: G_{329}



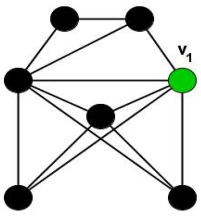
A.1.692: $G_{329} \setminus v_1$



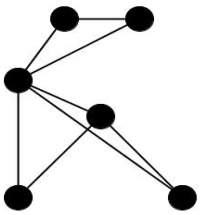
A.1.693: G_{330}



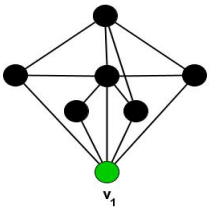
A.1.694: $G_{330} \setminus v_1$



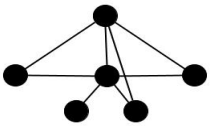
A.1.695: G_{331}



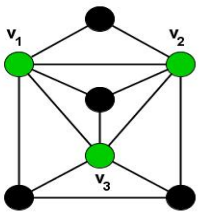
A.1.696: $G_{331} \setminus v_1$



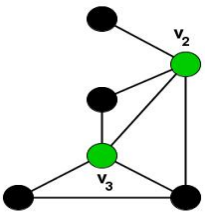
A.1.697: G_{332}



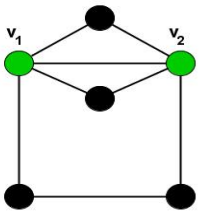
A.1.698: $G_{332} \setminus v_1$



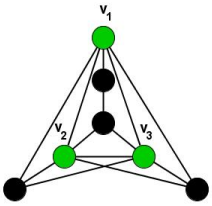
A.1.699: G_{333}



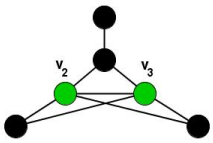
A.1.700: $G_{333} \setminus v_1$



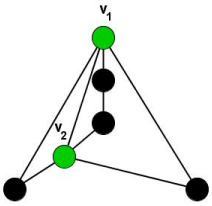
A.1.701: $G_{333} \setminus v_3$



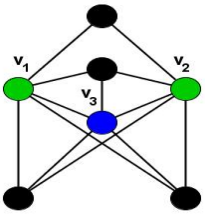
A.1.702: G_{334}



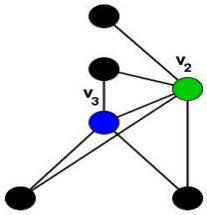
A.1.703: $G_{334} \setminus v_1$



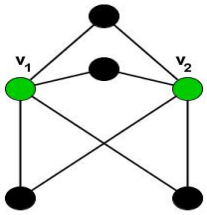
A.1.704: $G_{334} \setminus v_3$



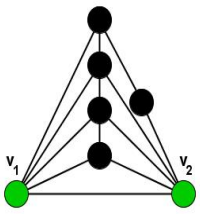
A.1.705: G_{335}



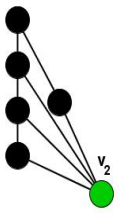
A.1.706: $G_{335} \setminus v_1$



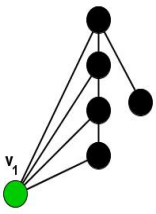
A.1.707: $G_{335} \setminus v_3$



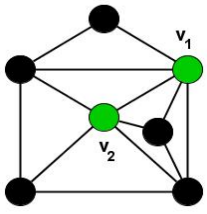
A.1.708: G_{336}



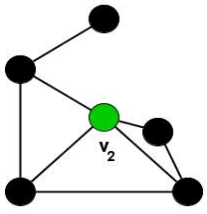
A.1.709: $G_{336} \setminus v_1$



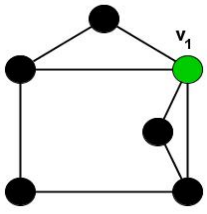
A.1.710: $G_{336} \setminus v_2$



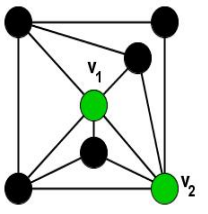
A.1.711: G_{337}



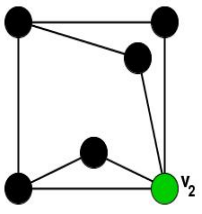
A.1.712: $G_{337} \setminus v_1$



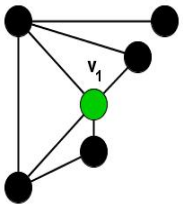
A.1.713: $G_{337} \setminus v_2$



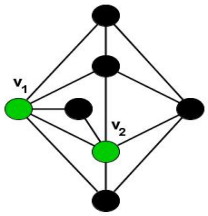
A.1.714: G_{338}



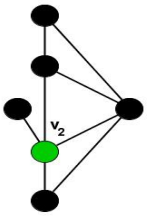
A.1.715: $G_{338} \setminus v_1$



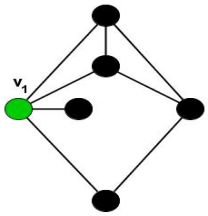
A.1.716: $G_{338} \setminus v_2$



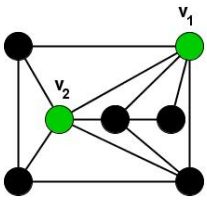
A.1.717: G_{339}



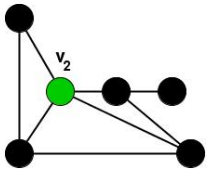
A.1.718: $G_{339} \setminus v_1$



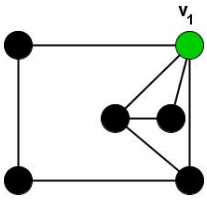
A.1.719: $G_{339} \setminus v_2$



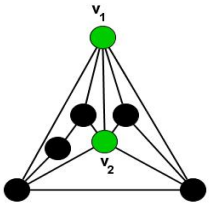
A.1.720: G_{340}



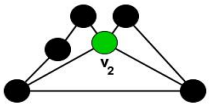
A.1.721: $G_{340} \setminus v_1$



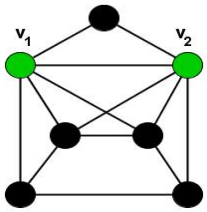
A.1.722: $G_{340} \setminus v_2$



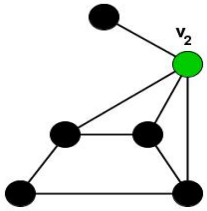
A.1.723: G_{341}



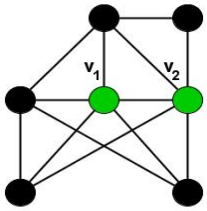
A.1.724: $G_{341} \setminus v_1$



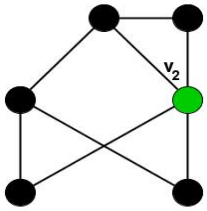
A.1.725: G_{342}



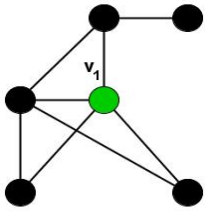
A.1.726: $G_{342} \setminus v_1$



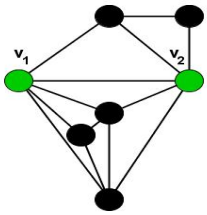
A.1.727: G_{343}



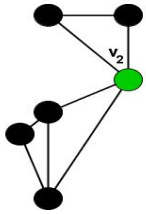
A.1.728: $G_{343} \setminus v_1$



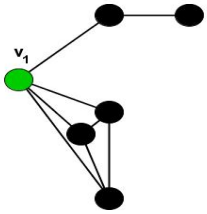
A.1.729: $G_{343} \setminus v_2$



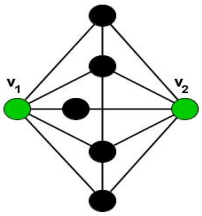
A.1.730: G_{344}



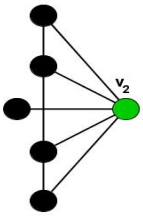
A.1.731: $G_{344} \setminus v_1$



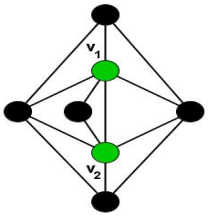
A.1.732: $G_{344} \setminus v_2$



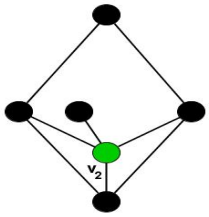
A.1.733: G_{345}



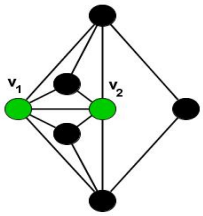
A.1.734: $G_{345} \setminus v_1$



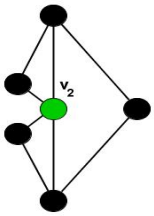
A.1.735: G_{346}



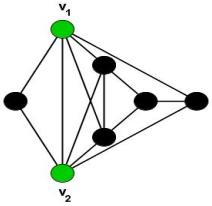
A.1.736: $G_{346} \setminus v_1$



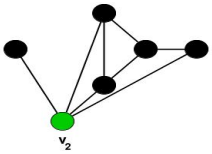
A.1.737: G_{347}



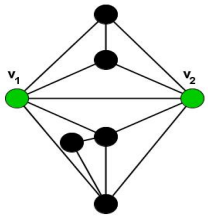
A.1.738: $G_{347} \setminus v_1$



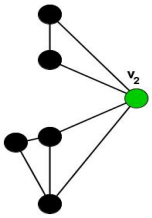
A.1.739: G_{348}



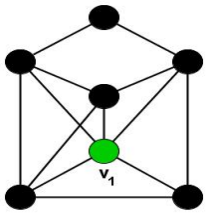
A.1.740: $G_{348} \setminus v_1$



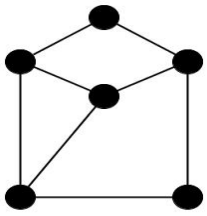
A.1.741: G_{349}



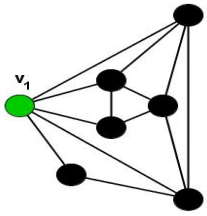
A.1.742: $G_{349} \setminus v_1$



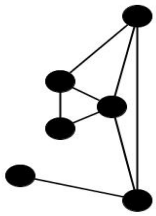
A.1.743: G_{350}



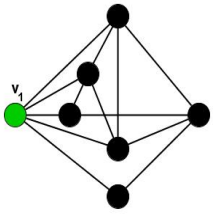
A.1.744: $G_{350} \setminus v_1$



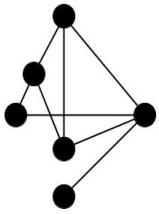
A.1.745: G_{351}



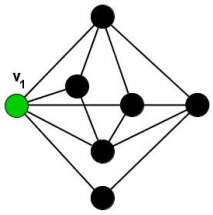
A.1.746: $G_{351} \setminus v_1$



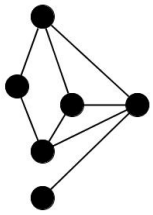
A.1.747: G_{352}



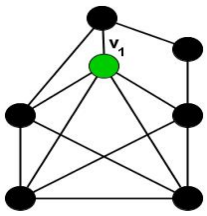
A.1.748: $G_{352} \setminus v_1$



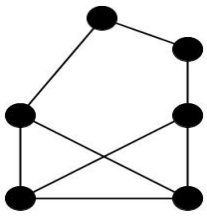
A.1.749: G_{353}



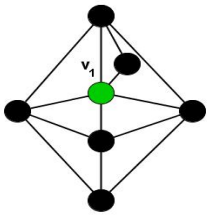
A.1.750: $G_{353} \setminus v_1$



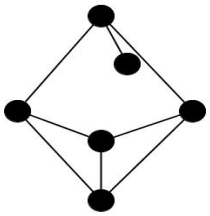
A.1.751: G_{354}



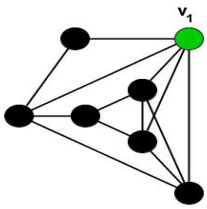
A.1.752: $G_{354} \setminus v_1$



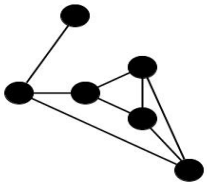
A.1.753: G_{355}



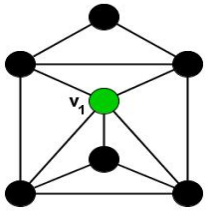
A.1.754: $G_{355} \setminus v_1$



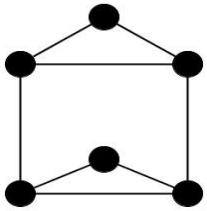
A.1.755: G_{356}



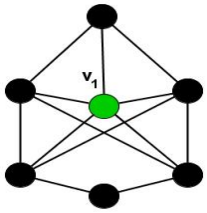
A.1.756: $G_{356} \setminus v_1$



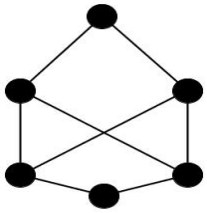
A.1.757: G_{357}



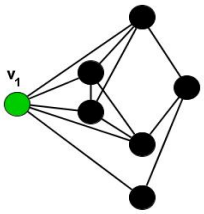
A.1.758: $G_{357} \setminus v_1$



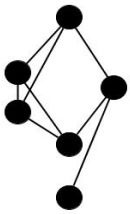
A.1.759: G_{358}



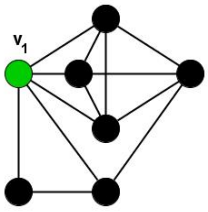
A.1.760: $G_{358} \setminus v_1$



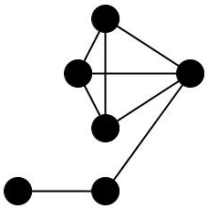
A.1.761: G_{359}



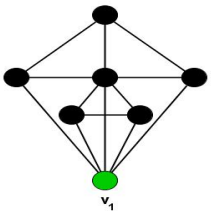
A.1.762: $G_{359} \setminus v_1$



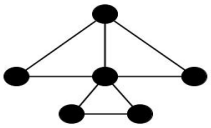
A.1.763: G_{360}



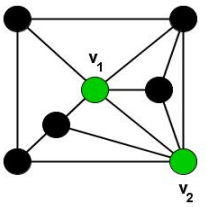
A.1.764: $G_{360} \setminus v_1$



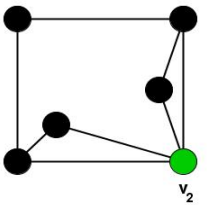
A.1.765: G_{361}



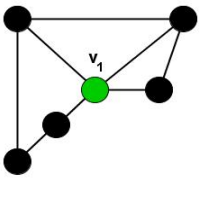
A.1.766: $G_{361} \setminus v_1$



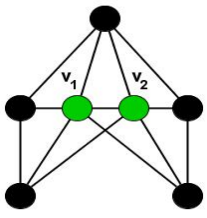
A.1.767: G_{362}



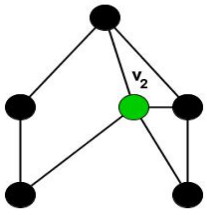
A.1.768: $G_{362} \setminus v_1$



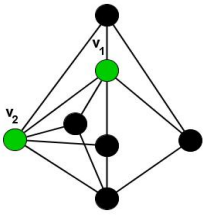
A.1.769: $G_{362} \setminus v_2$



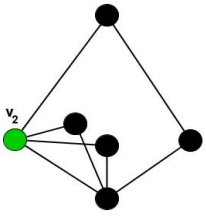
A.1.770: G_{363}



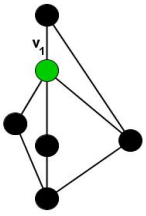
A.1.771: $G_{363} \setminus v_1$



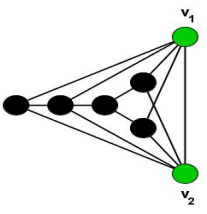
A.1.772: G_{364}



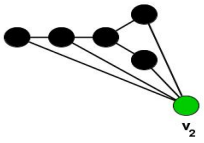
A.1.773: $G_{364} \setminus v_1$



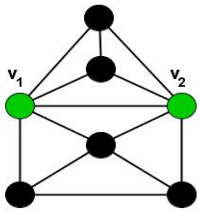
A.1.774: $G_{364} \setminus v_2$



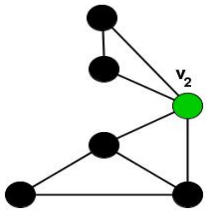
A.1.775: G_{365}



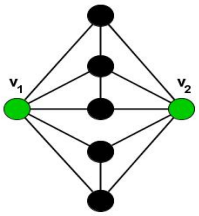
A.1.776: $G_{365} \setminus v_1$



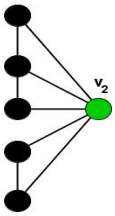
A.1.777: G_{366}



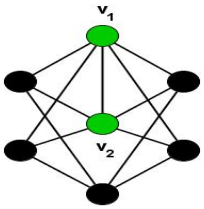
A.1.778: $G_{366} \setminus v_1$



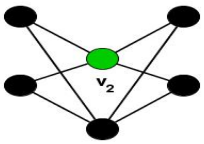
A.1.779: G_{367}



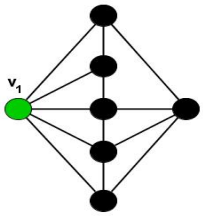
A.1.780: $G_{367} \setminus v_1$



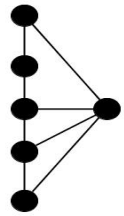
A.1.781: G_{368}



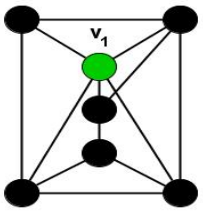
A.1.782: $G_{368} \setminus v_1$



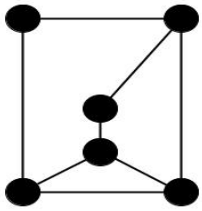
A.1.783: G_{369}



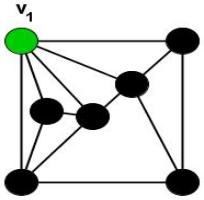
A.1.784: $G_{369} \setminus v_1$



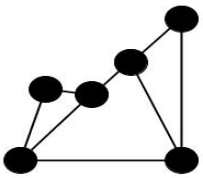
A.1.785: G_{370}



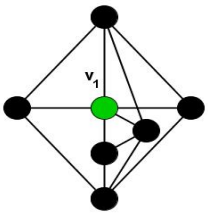
A.1.786: $G_{370} \setminus v_1$



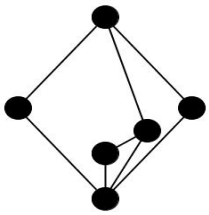
A.1.787: G_{371}



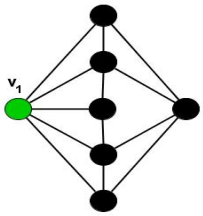
A.1.788: $G_{371} \setminus v_1$



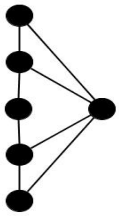
A.1.789: G_{372}



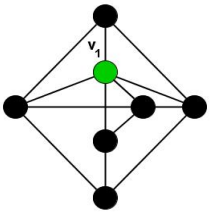
A.1.790: $G_{372} \setminus v_1$



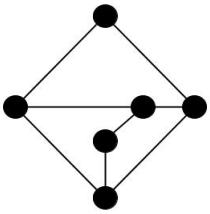
A.1.791: G_{373}



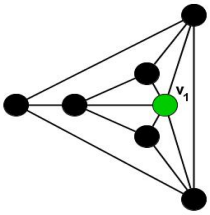
A.1.792: $G_{373} \setminus v_1$



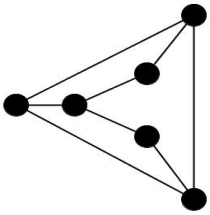
A.1.793: G_{374}



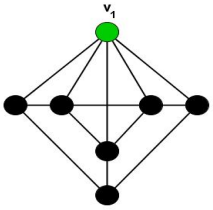
A.1.794: $G_{374} \setminus v_1$



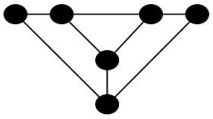
A.1.795: G_{375}



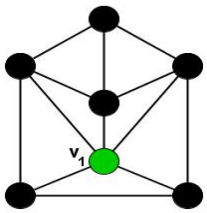
A.1.796: $G_{375} \setminus v_1$



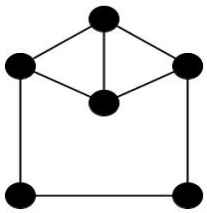
A.1.797: G_{376}



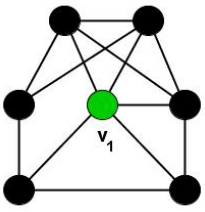
A.1.798: $G_{376} \setminus v_1$



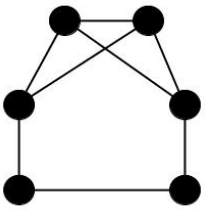
A.1.799: G_{377}



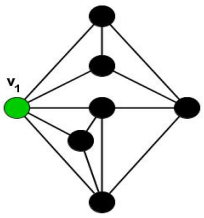
A.1.800: $G_{377} \setminus v_1$



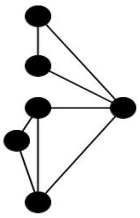
A.1.801: G_{378}



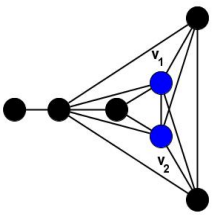
A.1.802: $G_{378} \setminus v_1$



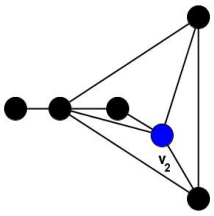
A.1.803: G_{379}



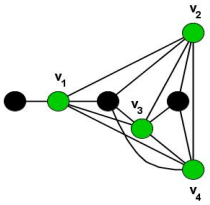
A.1.804: $G_{379} \setminus v_1$



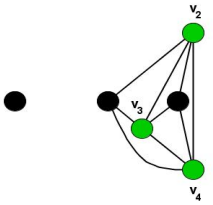
A.1.805: G_{380}



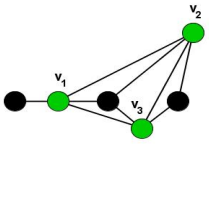
A.1.806: $G_{380} \setminus v_1$



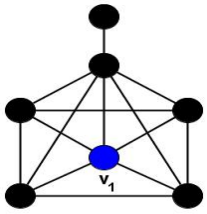
A.1.807: G_{381}



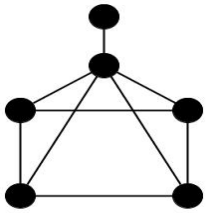
A.1.808: $G_{381} \setminus v_1$



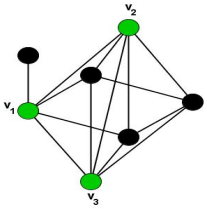
A.1.809: $G_{381} \setminus v_4$



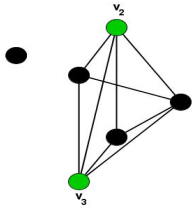
A.1.810: G_{382}



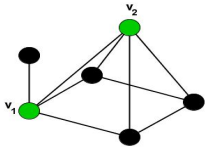
A.1.811: $G_{382} \setminus v_1$



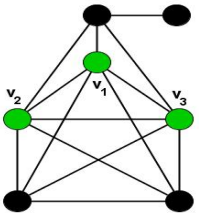
A.1.812: G_{383}



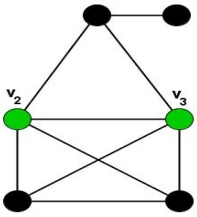
A.1.813: $G_{383} \setminus v_1$



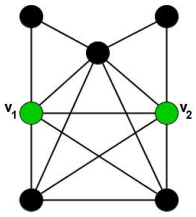
A.1.814: $G_{383} \setminus v_3$



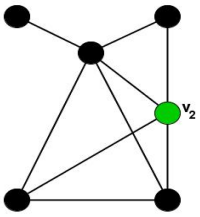
A.1.815: G_{384}



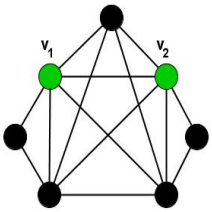
A.1.816: $G_{384} \setminus v_1$



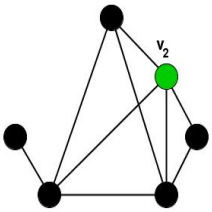
A.1.817: G_{385}



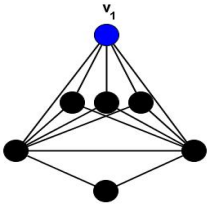
A.1.818: $G_{385} \setminus v_1$



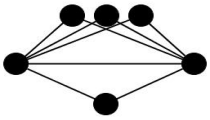
A.1.819: G_{386}



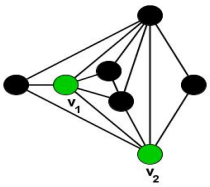
A.1.820: $G_{386} \setminus v_1$



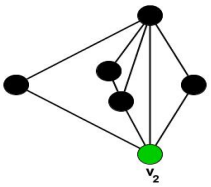
A.1.821: G_{387}



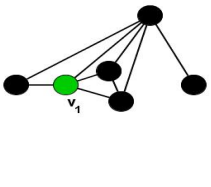
A.1.822: $G_{387} \setminus v_1$



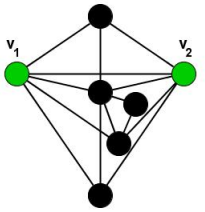
A.1.823: G_{388}



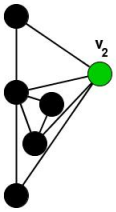
A.1.824: $G_{388} \setminus v_1$



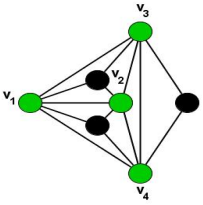
A.1.825: $G_{388} \setminus v_2$



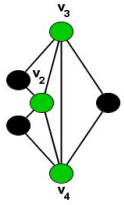
A.1.826: G_{389}



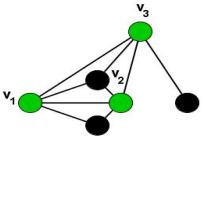
A.1.827: $G_{389} \setminus v_1$



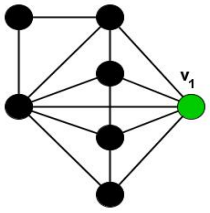
A.1.828: G_{390}



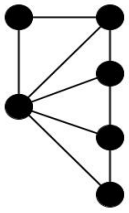
A.1.829: $G_{390} \setminus v_1$



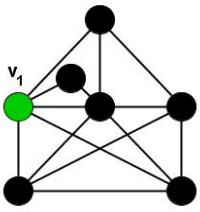
A.1.830: $G_{390} \setminus v_4$



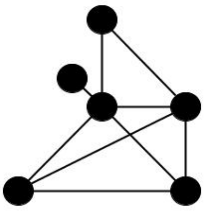
A.1.831: G_{391}



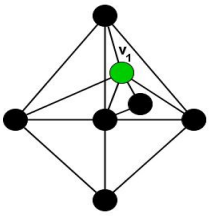
A.1.832: $G_{391} \setminus v_1$



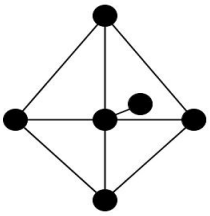
A.1.833: G_{392}



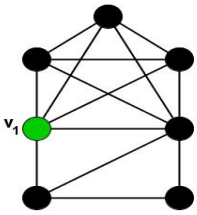
A.1.834: $G_{392} \setminus v_1$



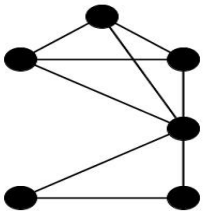
A.1.835: G_{393}



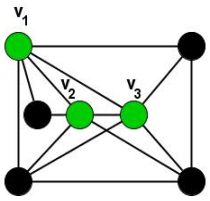
A.1.836: $G_{393} \setminus v_1$



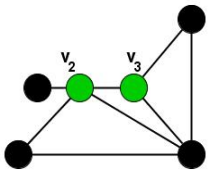
A.1.837: G_{394}



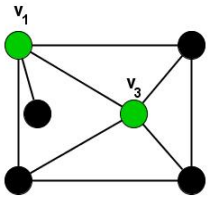
A.1.838: $G_{394} \setminus v_1$



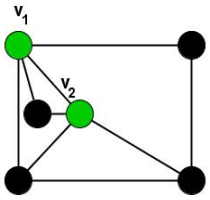
A.1.839: G_{395}



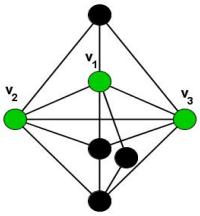
A.1.840: $G_{395} \setminus v_1$



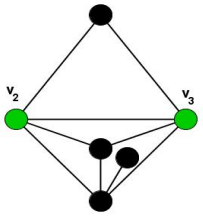
A.1.841: $G_{395} \setminus v_2$



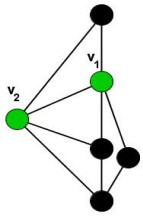
A.1.842: $G_{395} \setminus v_3$



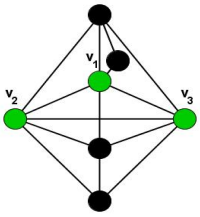
A.1.843: G_{396}



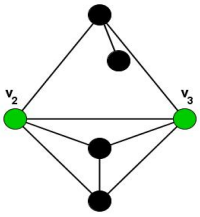
A.1.844: $G_{396} \setminus v_1$



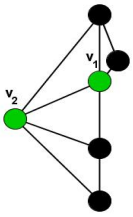
A.1.845: $G_{396} \setminus v_3$



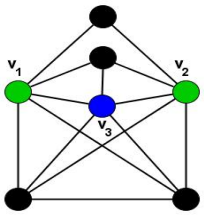
A.1.846: G_{397}



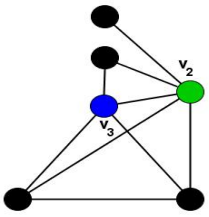
A.1.847: $G_{397} \setminus v_1$



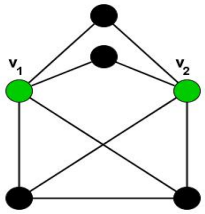
A.1.848: $G_{397} \setminus v_3$



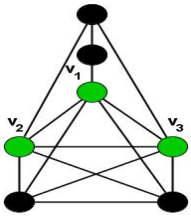
A.1.849: G_{398}



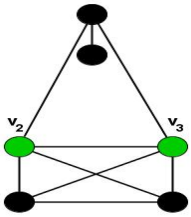
A.1.850: $G_{398} \setminus v_1$



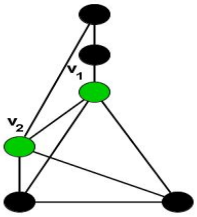
A.1.851: $G_{398} \setminus v_3$



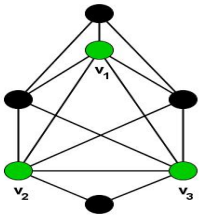
A.1.852: G_{399}



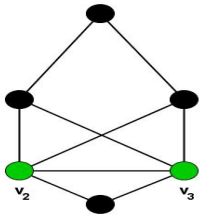
A.1.853: $G_{399} \setminus v_1$



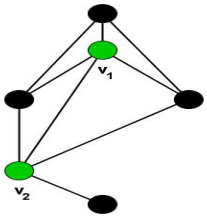
A.1.854: $G_{399} \setminus v_3$



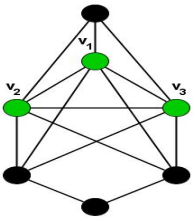
A.1.855: G_{400}



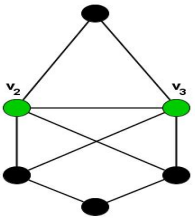
A.1.856: $G_{400} \setminus v_1$



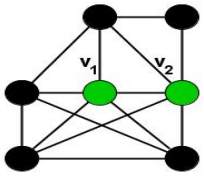
A.1.857: $G_{400} \setminus v_3$



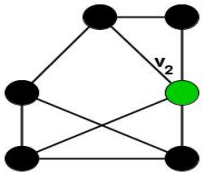
A.1.858: G_{401}



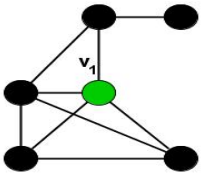
A.1.859: $G_{401} \setminus v_1$



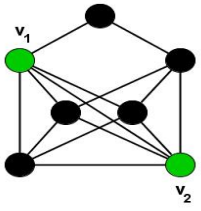
A.1.860: G_{402}



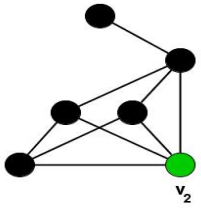
A.1.861: $G_{402} \setminus v_1$



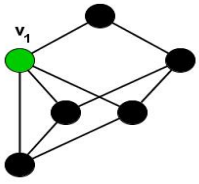
A.1.862: $G_{402} \setminus v_2$



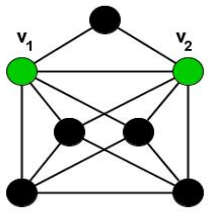
A.1.863: G_{403}



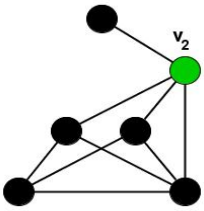
A.1.864: $G_{403} \setminus v_1$



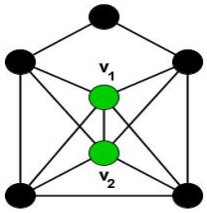
A.1.865: $G_{403} \setminus v_2$



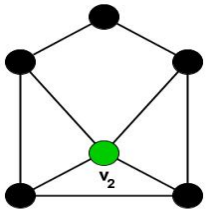
A.1.866: G_{404}



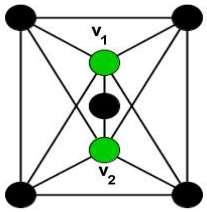
A.1.867: $G_{404} \setminus v_1$



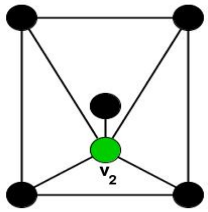
A.1.868: G_{405}



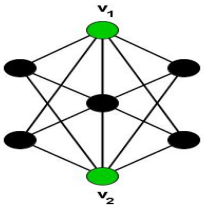
A.1.869: $G_{405} \setminus v_1$



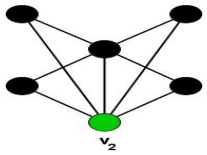
A.1.870: G_{406}



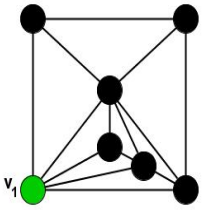
A.1.871: $G_{406} \setminus v_1$



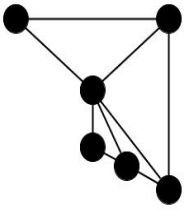
A.1.872: G_{407}



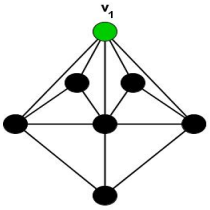
A.1.873: $G_{407} \setminus v_1$



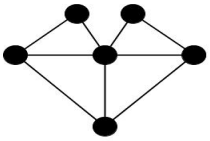
A.1.874: G_{408}



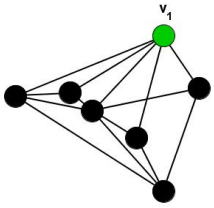
A.1.875: $G_{408} \setminus v_1$



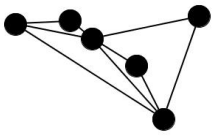
A.1.876: G_{409}



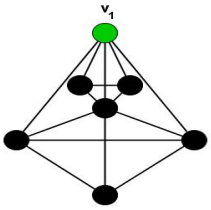
A.1.877: $G_{409} \setminus v_1$



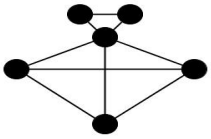
A.1.878: G_{410}



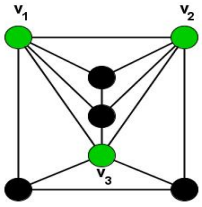
A.1.879: $G_{410} \setminus v_1$



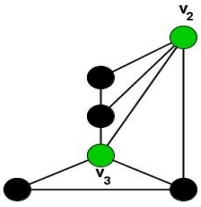
A.1.880: G_{411}



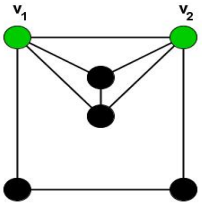
A.1.881: $G_{411} \setminus v_1$



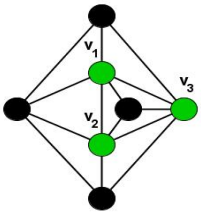
A.1.882: G_{412}



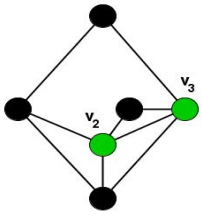
A.1.883: $G_{412} \setminus v_1$



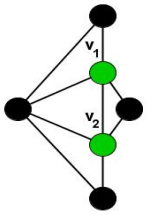
A.1.884: $G_{412} \setminus v_3$



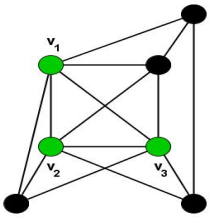
A.1.885: G_{413}



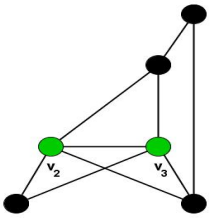
A.1.886: $G_{413} \setminus v_1$



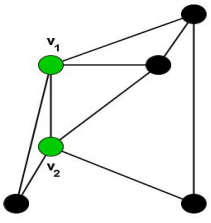
A.1.887: $G_{413} \setminus v_3$



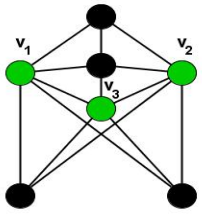
A.1.888: G_{414}



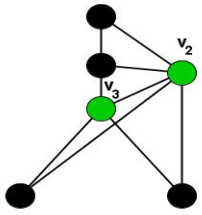
A.1.889: $G_{414} \setminus v_1$



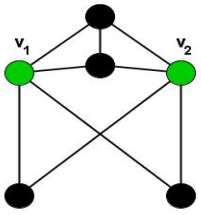
A.1.890: $G_{414} \setminus v_3$



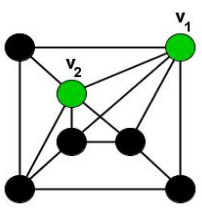
A.1.891: G_{415}



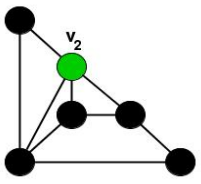
A.1.892: $G_{415} \setminus v_1$



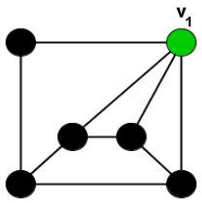
A.1.893: $G_{415} \setminus v_3$



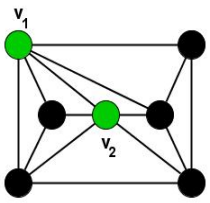
A.1.894: G_{416}



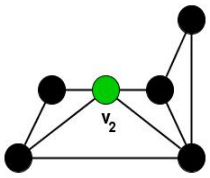
A.1.895: $G_{416} \setminus v_1$



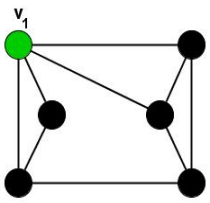
A.1.896: $G_{416} \setminus v_2$



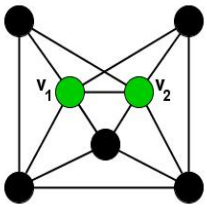
A.1.897: G_{417}



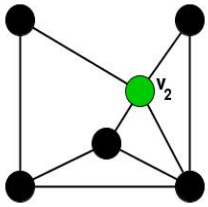
A.1.898: $G_{417} \setminus v_1$



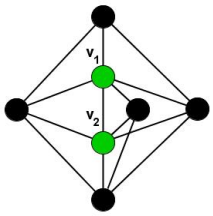
A.1.899: $G_{417} \setminus v_2$



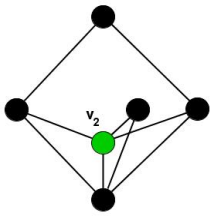
A.1.900: G_{418}



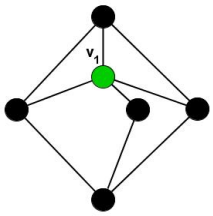
A.1.901: $G_{418} \setminus v_1$



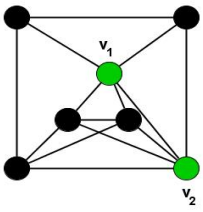
A.1.902: G_{419}



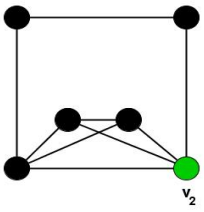
A.1.903: $G_{419} \setminus v_1$



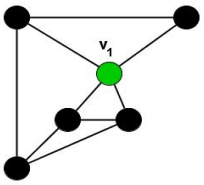
A.1.904: $G_{419} \setminus v_2$



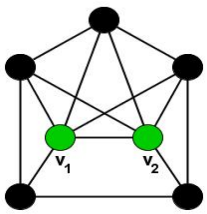
A.1.905: G_{420}



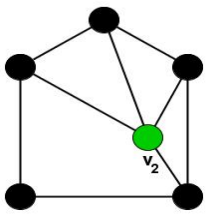
A.1.906: $G_{420} \setminus v_1$



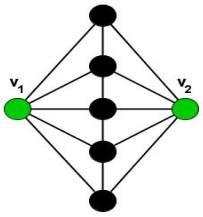
A.1.907: $G_{420} \setminus v_2$



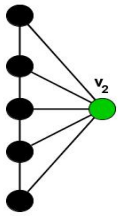
A.1.908: G_{421}



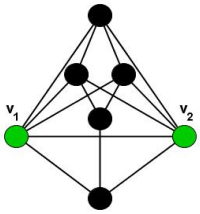
A.1.909: $G_{421} \setminus v_1$



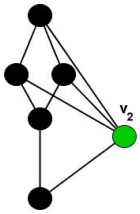
A.1.910: G_{422}



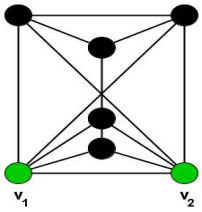
A.1.911: $G_{422} \setminus v_1$



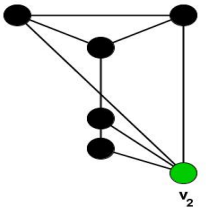
A.1.912: G_{423}



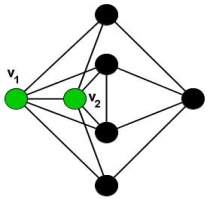
A.1.913: $G_{423} \setminus v_1$



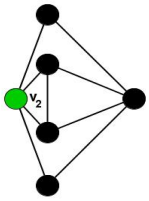
A.1.914: G_{424}



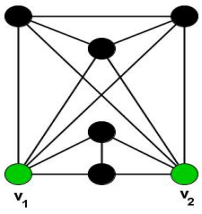
A.1.915: $G_{424} \setminus v_1$



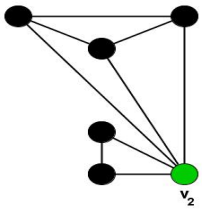
A.1.916: G_{425}



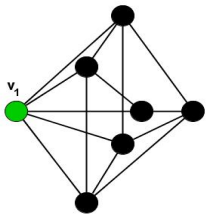
A.1.917: $G_{425} \setminus v_1$



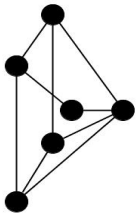
A.1.918: G_{426}



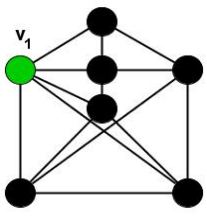
A.1.919: $G_{426} \setminus v_1$



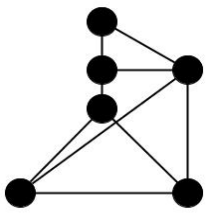
A.1.920: G_{427}



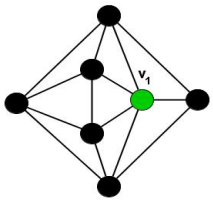
A.1.921: $G_{427} \setminus v_1$



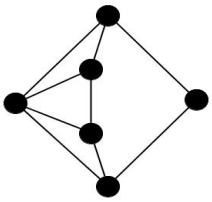
A.1.922: G_{428}



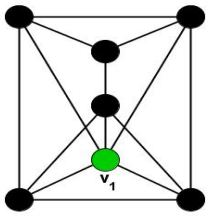
A.1.923: $G_{428} \setminus v_1$



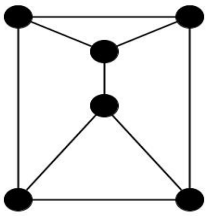
A.1.924: G_{429}



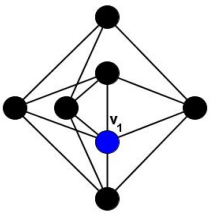
A.1.925: $G_{429} \setminus v_1$



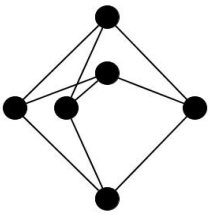
A.1.926: G_{430}



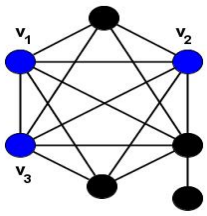
A.1.927: $G_{430} \setminus v_1$



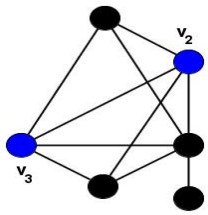
A.1.928: G_{431}



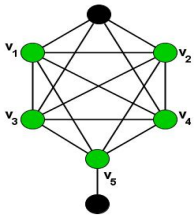
A.1.929: $G_{431} \setminus v_1$



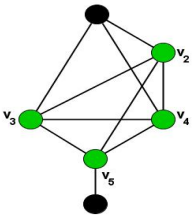
A.1.930: G_{432}



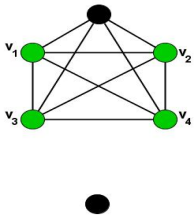
A.1.931: $G_{432} \setminus v_1$



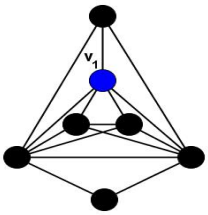
A.1.932: G_{433}



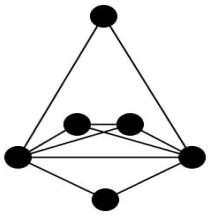
A.1.933: $G_{433} \setminus v_1$



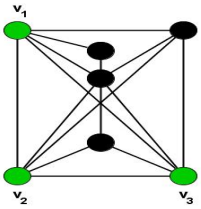
A.1.934: $G_{433} \setminus v_5$



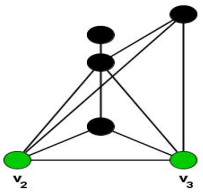
A.1.935: G_{434}



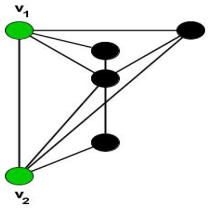
A.1.936: $G_{434} \setminus v_1$



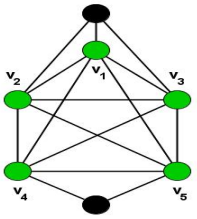
A.1.937: G_{435}



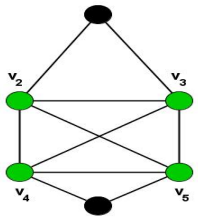
A.1.938: $G_{435} \setminus v_1$



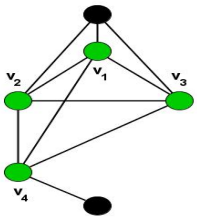
A.1.939: $G_{435} \setminus v_3$



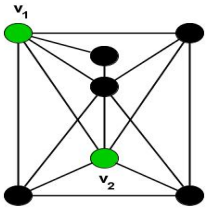
A.1.940: G_{436}



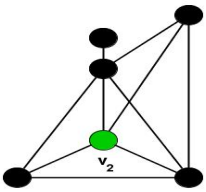
A.1.941: $G_{436} \setminus v_1$



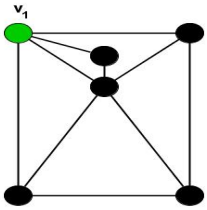
A.1.942: $G_{436} \setminus v_5$



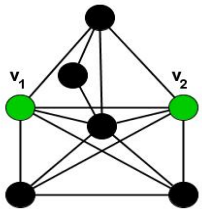
A.1.943: G_{437}



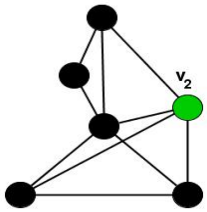
A.1.944: $G_{437} \setminus v_1$



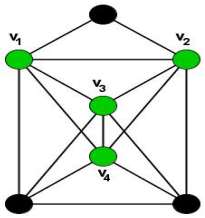
A.1.945: $G_{437} \setminus v_2$



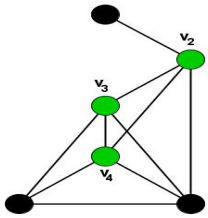
A.1.946: G_{438}



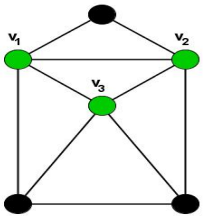
A.1.947: $G_{438} \setminus v_1$



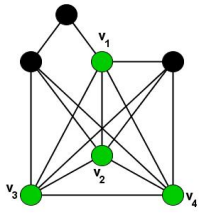
A.1.948: G_{439}



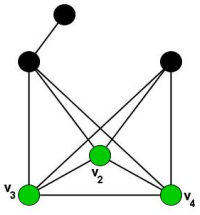
A.1.949: $G_{439} \setminus v_1$



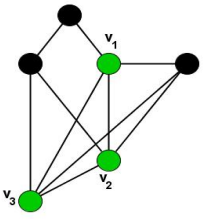
A.1.950: $G_{439} \setminus v_4$



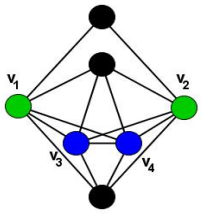
A.1.951: G_{440}



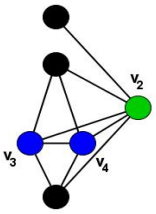
A.1.952: $G_{440} \setminus v_1$



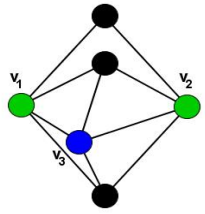
A.1.953: $G_{440} \setminus v_4$



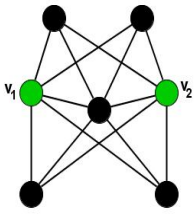
A.1.954: G_{441}



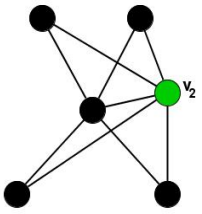
A.1.955: $G_{441} \setminus v_1$



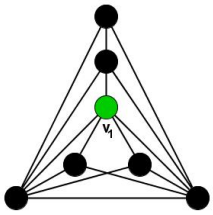
A.1.956: $G_{441} \setminus v_4$



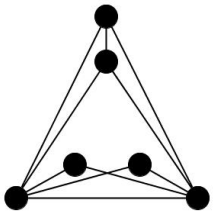
A.1.957: G_{442}



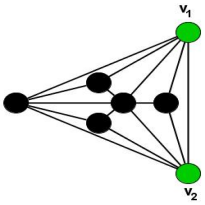
A.1.958: $G_{442} \setminus v_1$



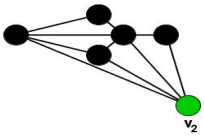
A.1.959: G_{443}



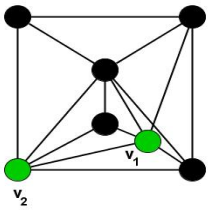
A.1.960: $G_{443} \setminus v_1$



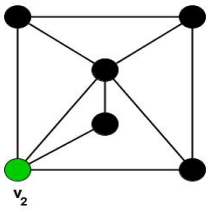
A.1.961: G_{444}



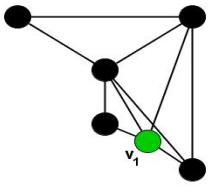
A.1.962: $G_{444} \setminus v_1$



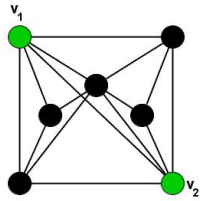
A.1.963: G_{445}



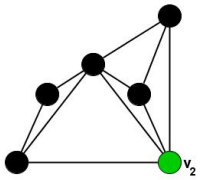
A.1.964: $G_{445} \setminus v_1$



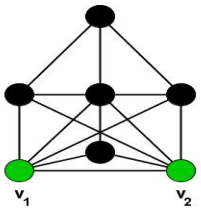
A.1.965: $G_{445} \setminus v_2$



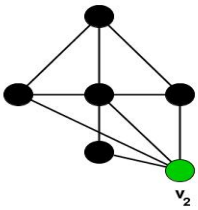
A.1.966: G_{446}



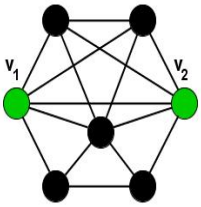
A.1.967: $G_{446} \setminus v_1$



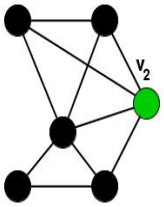
A.1.968: G_{447}



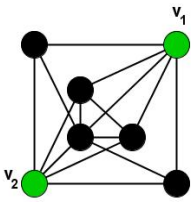
A.1.969: $G_{447} \setminus v_1$



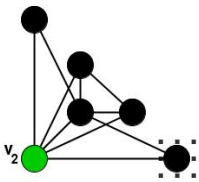
A.1.970: G_{448}



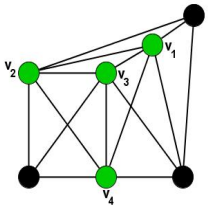
A.1.971: $G_{448} \setminus v_1$



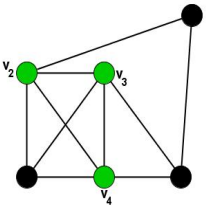
A.1.972: G_{449}



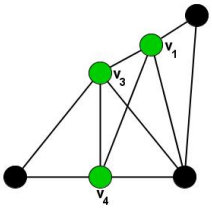
A.1.973: $G_{449} \setminus v_1$



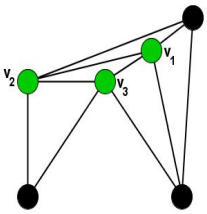
A.1.974: G_{450}



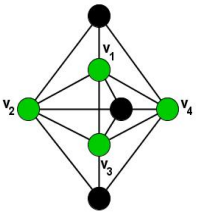
A.1.975: $G_{450} \setminus v_1$



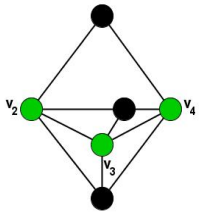
A.1.976: $G_{450} \setminus v_2$



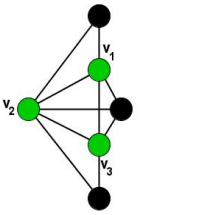
A.1.977: $G_{450} \setminus v_4$



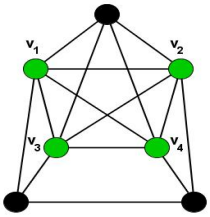
A.1.978: G_{451}



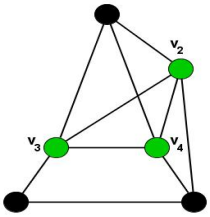
A.1.979: $G_{451} \setminus v_1$



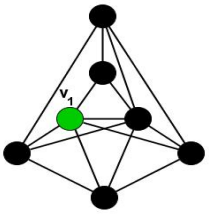
A.1.980: $G_{451} \setminus v_4$



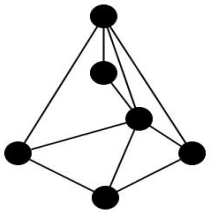
A.1.981: G_{452}



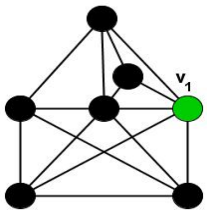
A.1.982: $G_{452} \setminus v_1$



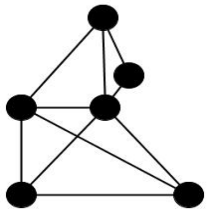
A.1.983: G_{453}



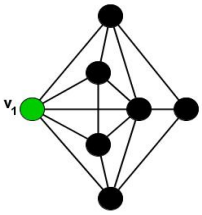
A.1.984: $G_{453} \setminus v_1$



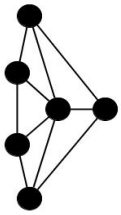
A.1.985: G_{454}



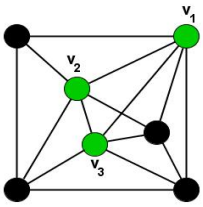
A.1.986: $G_{454} \setminus v_1$



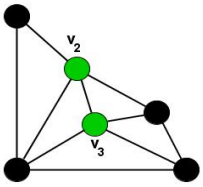
A.1.987: G_{455}



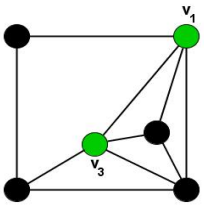
A.1.988: $G_{455} \setminus v_1$



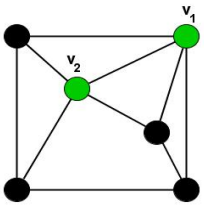
A.1.989: G_{456}



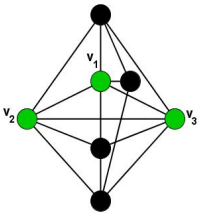
A.1.990: $G_{456} \setminus v_1$



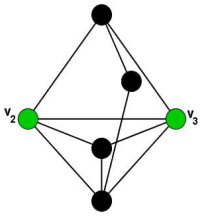
A.1.991: $G_{456} \setminus v_2$



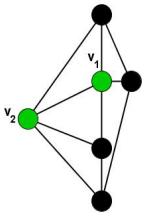
A.1.992: $G_{456} \setminus v_3$



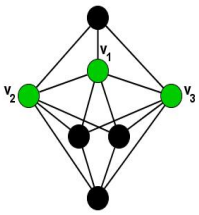
A.1.993: G_{457}



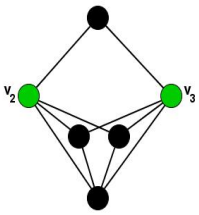
A.1.994: $G_{457} \setminus v_1$



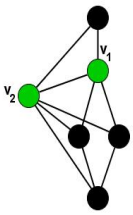
A.1.995: $G_{457} \setminus v_3$



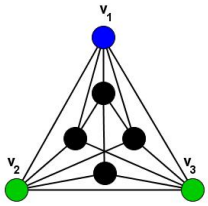
A.1.996: G_{458}



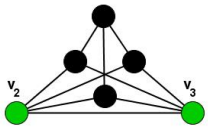
A.1.997: $G_{458} \setminus v_1$



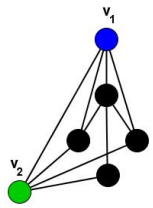
A.1.998: $G_{458} \setminus v_3$



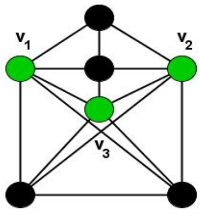
A.1.999: G_{459}



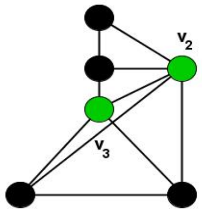
A.1.1000: $G_{459} \setminus v_1$



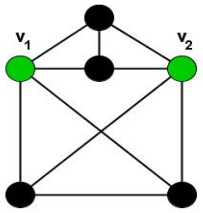
A.1.1001: $G_{459} \setminus v_3$



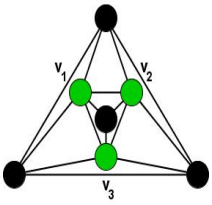
A.1.1002: G_{460}



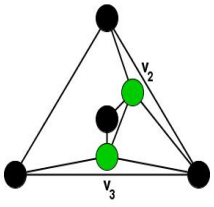
A.1.1003: $G_{460} \setminus v_1$



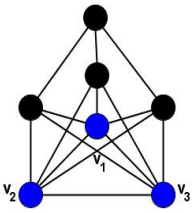
A.1.1004: $G_{460} \setminus v_3$



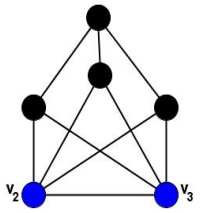
A.1.1005: G_{461}



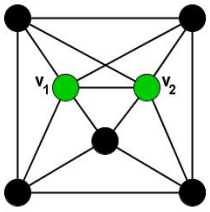
A.1.1006: $G_{461} \setminus v_1$



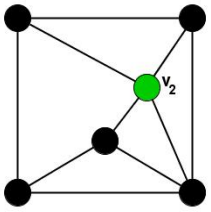
A.1.1007: G_{462}



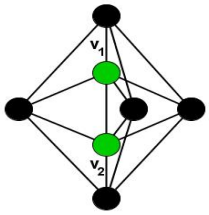
A.1.1008: $G_{462} \setminus v_1$



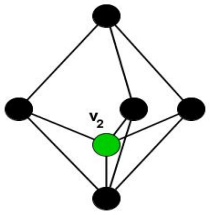
A.1.1009: G_{463}



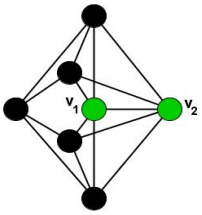
A.1.1010: $G_{463} \setminus v_1$



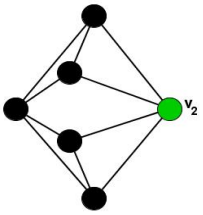
A.1.1011: G_{464}



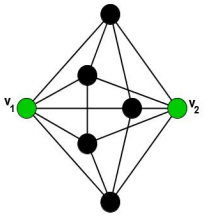
A.1.1012: $G_{464} \setminus v_1$



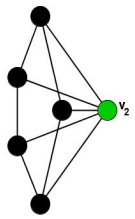
A.1.1013: G_{465}



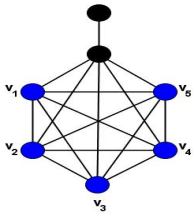
A.1.1014: $G_{465} \setminus v_1$



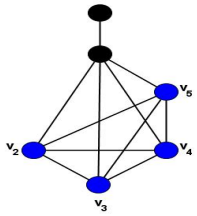
A.1.1015: G_{466}



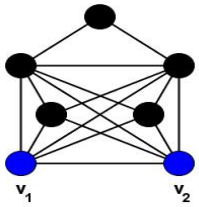
A.1.1016: $G_{466} \setminus v_1$



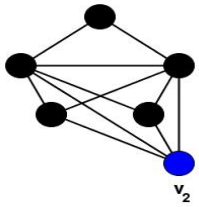
A.1.1017: G_{467}



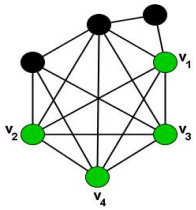
A.1.1018: $G_{467} \setminus v_1$



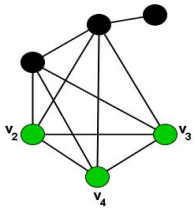
A.1.1019: G_{468}



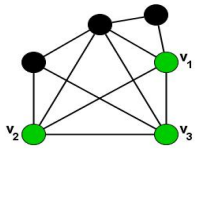
A.1.1020: $G_{468} \setminus v_1$



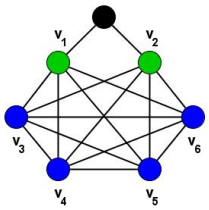
A.1.1021: G_{469}



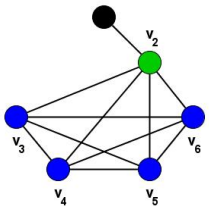
A.1.1022: $G_{469} \setminus v_1$



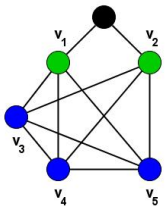
A.1.1023: $G_{469} \setminus v_4$



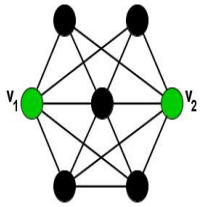
A.1.1024: G_{470}



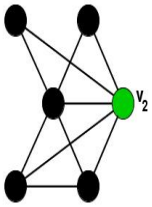
A.1.1025: $G_{470} \setminus v_1$



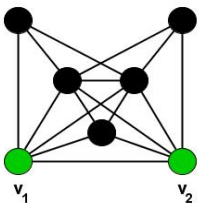
A.1.1026: $G_{470} \setminus v_6$



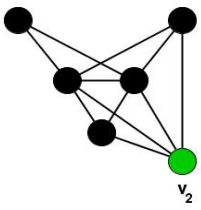
A.1.1027: G_{471}



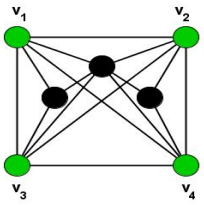
A.1.1028: $G_{471} \setminus v_1$



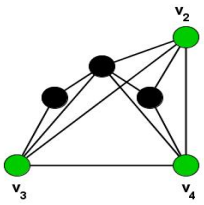
A.1.1029: G_{472}



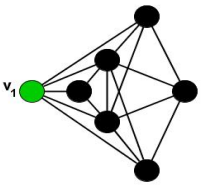
A.1.1030: $G_{472} \setminus v_1$



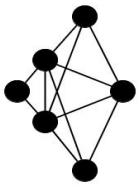
A.1.1031: G_{473}



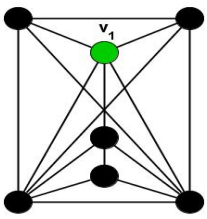
A.1.1032: $G_{473} \setminus v_1$



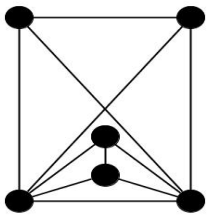
A.1.1033: G_{474}



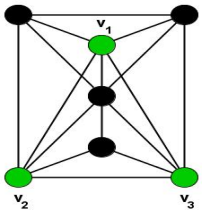
A.1.1034: $G_{474} \setminus v_1$



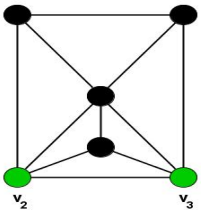
A.1.1035: G_{475}



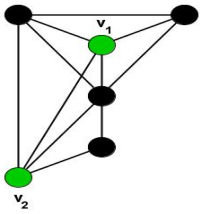
A.1.1036: $G_{475} \setminus v_1$



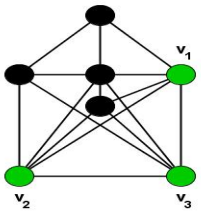
A.1.1037: G_{476}



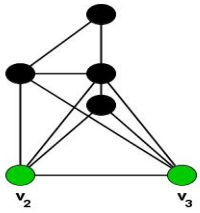
A.1.1038: $G_{476} \setminus v_1$



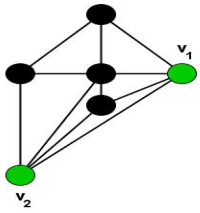
A.1.1039: $G_{476} \setminus v_3$



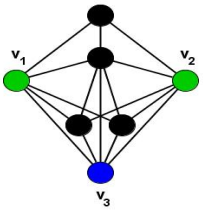
A.1.1040: G_{477}



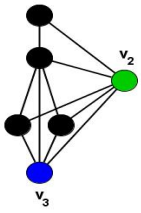
A.1.1041: $G_{477} \setminus v_1$



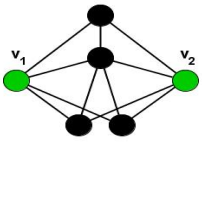
A.1.1042: $G_{477} \setminus v_3$



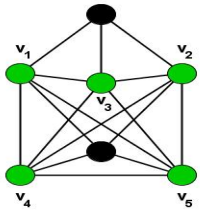
A.1.1043: G_{478}



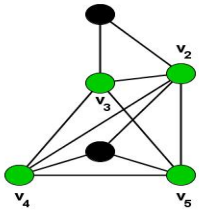
A.1.1044: $G_{478} \setminus v_1$



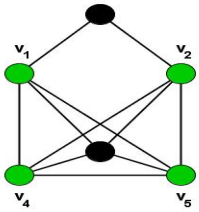
A.1.1045: $G_{478} \setminus v_3$



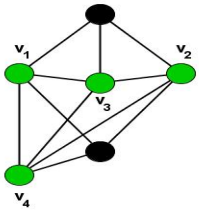
A.1.1046: G_{479}



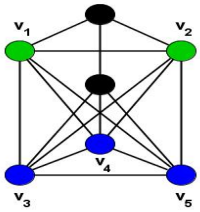
A.1.1047: $G_{479} \setminus v_1$



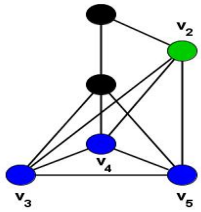
A.1.1048: $G_{479} \setminus v_3$



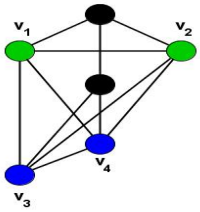
A.1.1049: $G_{479} \setminus v_5$



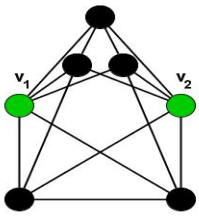
A.1.1050: G_{480}



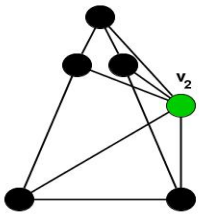
A.1.1051: $G_{480} \setminus v_1$



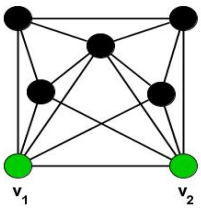
A.1.1052: $G_{480} \setminus v_5$



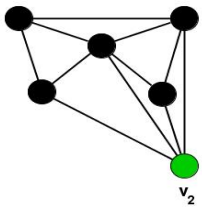
A.1.1053: G_{481}



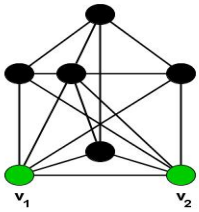
A.1.1054: $G_{481} \setminus v_1$



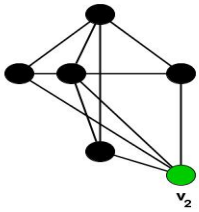
A.1.1055: G_{482}



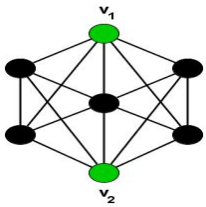
A.1.1056: $G_{482} \setminus v_1$



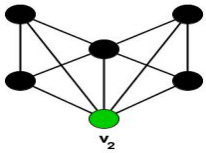
A.1.1057: G_{483}



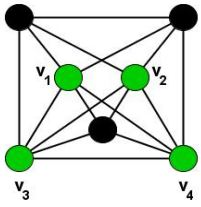
A.1.1058: $G_{483} \setminus v_1$



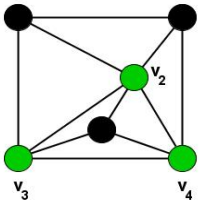
A.1.1059: G_{484}



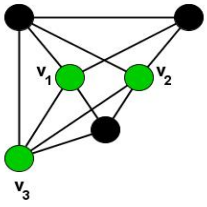
A.1.1060: $G_{484} \setminus v_1$



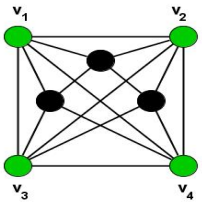
A.1.1061: G_{485}



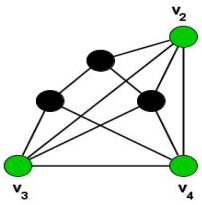
A.1.1062: $G_{485} \setminus v_1$



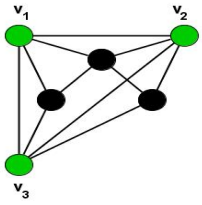
A.1.1063: $G_{485} \setminus v_4$



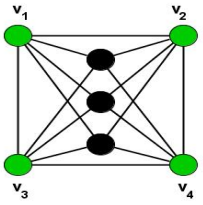
A.1.1064: G_{486}



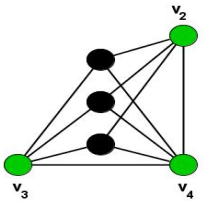
A.1.1065: $G_{486} \setminus v_1$



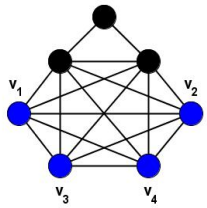
A.1.1066: $G_{486} \setminus v_4$



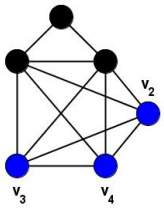
A.1.1067: G_{487}



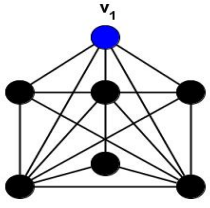
A.1.1068: $G_{487} \setminus v_1$



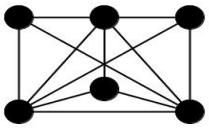
A.1.1069: G_{488}



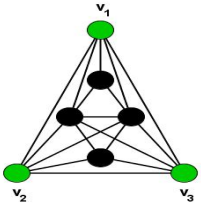
A.1.1070: $G_{488} \setminus v_1$



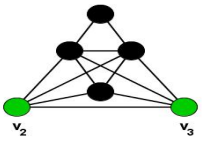
A.1.1071: G_{489}



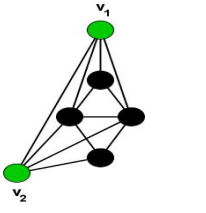
A.1.1072: $G_{489} \setminus v_1$



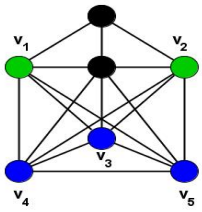
A.1.1073: G_{490}



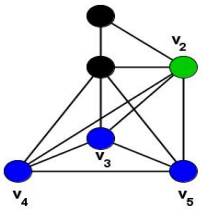
A.1.1074: $G_{490} \setminus v_1$



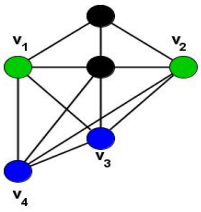
A.1.1075: $G_{490} \setminus v_3$



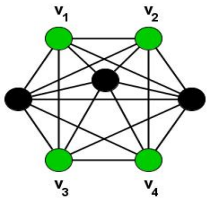
A.1.1076: G_{491}



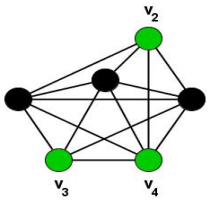
A.1.1077: $G_{491} \setminus v_1$



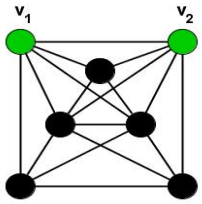
A.1.1078: $G_{491} \setminus v_5$



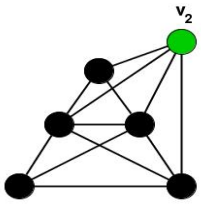
A.1.1079: G_{492}



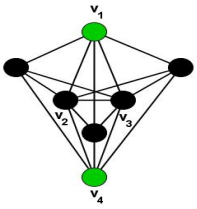
A.1.1080: $G_{492} \setminus v_1$



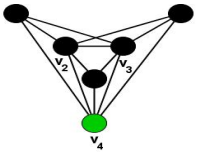
A.1.1081: G_{493}



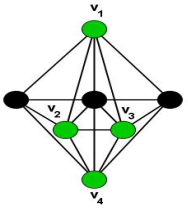
A.1.1082: $G_{493} \setminus v_1$



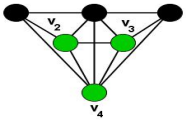
A.1.1083: G_{494}



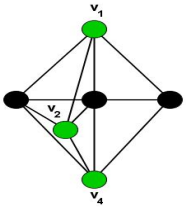
A.1.1084: $G_{494} \setminus v_1$



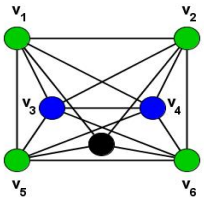
A.1.1085: G_{495}



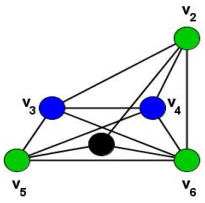
A.1.1086: $G_{495} \setminus v_1$



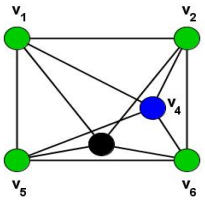
A.1.1087: $G_{495} \setminus v_3$



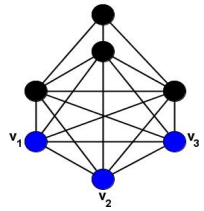
A.1.1088: G_{496}



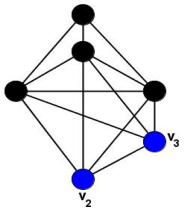
A.1.1089: $G_{496} \setminus v_1$



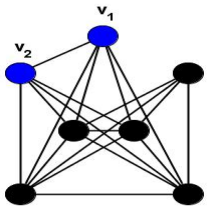
A.1.1090: $G_{496} \setminus v_3$



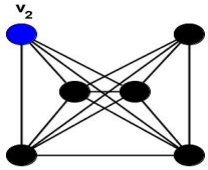
A.1.1091: G_{497}



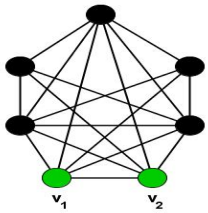
A.1.1092: $G_{497} \setminus v_1$



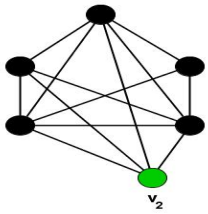
A.1.1093: G_{498}



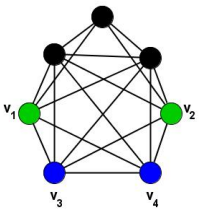
A.1.1094: $G_{498} \setminus v_1$



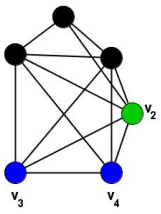
A.1.1095: G_{499}



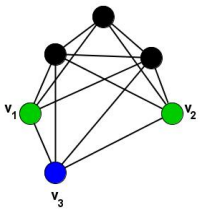
A.1.1096: $G_{499} \setminus v_1$



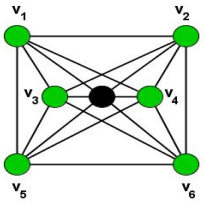
A.1.1097: G_{500}



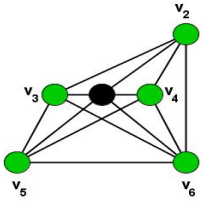
A.1.1098: $G_{500} \setminus v_1$



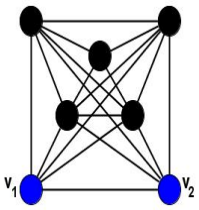
A.1.1099: $G_{500} \setminus v_4$



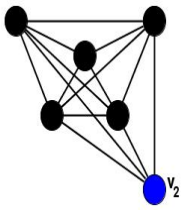
A.1.1100: G_{501}



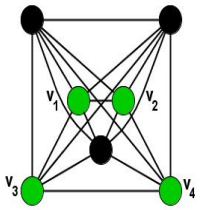
A.1.1101: $G_{501} \setminus v_1$



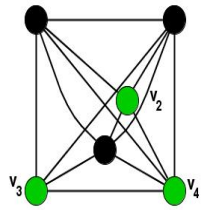
A.1.1102: G_{502}



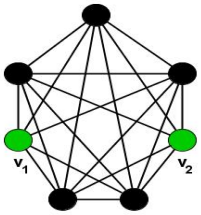
A.1.1103: $G_{502} \setminus v_1$



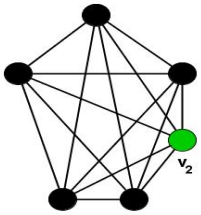
A.1.1104: G_{503}



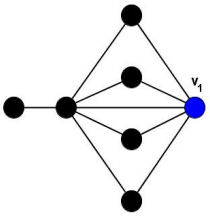
A.1.1105: $G_{503} \setminus v_1$



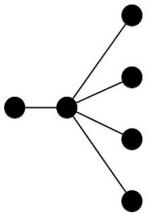
A.1.1106: G_{504}



A.1.1107: $G_{504} \setminus v_1$



A.1.1108: G_{505}



A.1.1109: $G_{505} \setminus v_1$

Apêndice B - Análise da função de perturbação para grafos com n vértices, $4 \leq n \leq 7$, e m arestas, $3 \leq m \leq 21$, que possuem vértices de grau $(n-2)$

Tabela B.1: Grafos com 4 vértices

$Grafo$	$Spec$	$\phi(v)$
G_1	$(3.41\ 2\ 0.59\ 0)$	
$G_1 \setminus v_1$	$(2\ 0^2)$	0.59
G_2	$(4\ 3\ 1\ 0)$	
$G_2 \setminus v_1$	$(3\ 1\ 0)$	0
G_3	$(4\ 2^2\ 0)$	
$G_3 \setminus v_1$	$(3\ 1\ 0)$	1
G_4	$(4^2\ 2\ 0)$	
$G_4 \setminus v_1$	$(3^2\ 0)$	-1

Tabela B.2: Grafos com 5 vértices

$Grafo$	$Spec$	$\phi(v)$
G_5	$(4.17\ 2.31\ 1\ 0.52\ 0)$	
$G_5 \setminus v_1$	$(2\ 0^3)$	0.52
G_6	$(4.30\ 3.62\ 1.38\ 0.70\ 0)$	
$G_6 \setminus v_1$	$(3\ 1\ 0^2)$	0.70
G_7	$(4.17\ 3\ 2.31\ 0.52\ 0)$	
$G_7 \setminus v_1$	$(2^2\ 0^2)$	0.52
G_8	$(4.48\ 2.69\ 2\ 0.83\ 0)$	
$G_8 \setminus v_1$	$(3\ 1\ 0^2)$	0.83
G_9	$(5\ 4\ 2\ 1\ 0)$	
$G_9 \setminus v_1$	$(4\ 1^2\ 0)$	0
G_{10}	$(4.48\ 4\ 2.69\ 0.83\ 0)$	
$G_{10} \setminus v_1$	$(3^2\ 0^2)$	0.83
$G_{10} \setminus v_3$	$(3.41\ 2\ 0.59\ 0)$	0.24
G_{11}	$(4.62\ 3.62\ 2.38\ 1.38\ 0)$	
$G_{11} \setminus v_1$	$(3.41\ 2\ 0.59\ 0)$	0.79
G_{12}	$(5\ 3\ 2^2\ 0)$	
$G_{12} \setminus v_1$	$(4\ 1^2\ 0)$	1
G_{13}	$(5\ 4^2\ 1\ 0)$	

$G_{13} \setminus v_1$	(4 3 1 0)	0
G_{14}	(5 4.41 3 1.59 0)	
$G_{14} \setminus v_1$	(4 3 1 0)	0.59
G_{15}	(5 4 3 2 0)	
$G_{15} \setminus v_1$	(4 2^2 0)	0
G_{16}	(5^2 4 2 0)	
$G_{16} \setminus v_1$	(4^2 2 0)	0
G_{17}	(5^2 3^2 0)	
$G_{17} \setminus v_1$	(4^2 2 0)	1
G_{18}	(5^3 3 0)	
$G_{18} \setminus v_1$	(4^3 0)	-1

Tabela B.3: Grafos com 6 vértices

$Grafo$	$Spec$	$\phi(v)$
G_{19}	(5.09 2.43 1^2 0.49 0)	
$G_{19} \setminus v_1$	(2 0^4)	0.49
G_{20}	(5.11 3.79 1.47 1 0.63 0)	
$G_{20} \setminus v_1$	(3 1 0^3)	0.63
G_{21}	(5.09 3 2.43 1 0.49 0)	
$G_{21} \setminus v_1$	(2^2 0^3)	0.49
G_{22}	(5.24 3 2 1 0.76 0)	
$G_{22} \setminus v_1$	(3 1 0^3)	0.76
G_{23}	(5.24 4.73 2 1.27 0.76 0)	
$G_{23} \setminus v_1$	(4 1^2 0^2)	0.76
G_{24}	(5.11 4.30 2.75 1.14 0.70 0)	
$G_{24} \setminus v_1$	(3.41 2 0.59 0^2)	0.70
G_{25}	(5.24 4 3 1 0.76 0)	
$G_{25} \setminus v_1$	(3^2 0^3)	0.76
G_{26}	(5.27 3.86 2.53 1.45 0.88 0)	
$G_{26} \setminus v_1$	(3.41 2 0.59 0^2)	0.88
G_{27}	(5.11 3.79 3 1.47 0.63 0)	
$G_{27} \setminus v_1$	(3 2 1 0^2)	0.63
G_{28}	(5.09 4 2.43 2 0.49 0)	
$G_{28} \setminus v_1$	(3 2 1 0^2)	0.49
G_{29}	(5.51 3.57 2^2 0.91 0)	
$G_{29} \setminus v_1$	(4 1^2 0^2)	0.91
G_{30}	(5.24 3^2 2 0.76 0)	

$G_{30} \setminus v_1$	$(3\ 2\ 1\ 0^2)$	0.76
G_{31}	(5.24 4.73 4 1.27 0.76 0)	
$G_{31} \setminus v_1$	$(4\ 3\ 1\ 0^2)$	0.76
G_{32}	$(6\ 5\ 2^2\ 1\ 0)$	
$G_{32} \setminus v_1$	$(5\ 1^4\ 0)$	0
G_{33}	(5.30 4.86 3.25 1.70 0.89 0)	
$G_{33} \setminus v_1$	(4.17 2.31 1 0.52 0)	0.37
$G_{33} \setminus v_2$	$(4\ 3\ 1\ 0^2)$	0.89
G_{34}	(5.24 5 3 2 0.76 0)	
$G_{34} \setminus v_1$	(4.17 2.31 1 0.52 0)	0.24
G_{35}	(5.12 4.55 3.47 2.14 0.73 0)	
$G_{35} \setminus v_1$	(3.62 2.62 1.38 0.38 0)	0.35
G_{36}	(5.37 4.53 3 2.21 0.89 0)	
$G_{36} \setminus v_1$	$(4\ 2^2\ 0^2)$	0.89
G_{37}	(5.51 4 3.57 2 0.91 0)	
$G_{37} \setminus v_1$	$(4\ 3\ 1\ 0^2)$	0.91
G_{38}	$(5.09\ 4^2\ 2.43\ 0.49\ 0)$	
$G_{38} \setminus v_1$	$(3^2\ 2\ 0^2)$	0.49
G_{39}	(5.41 4.73 2.59 2 1.27 0)	
$G_{39} \setminus v_1$	(4.17 2.31 1 0.52 0)	0.75
G_{40}	$(6\ 4\ 2^3\ 0)$	
$G_{40} \setminus v_1$	$(5\ 1^3\ 0)$	1
G_{41}	(5.28 4.32 3 2.30 1.11 0)	
$G_{41} \setminus v_1$	(3.62 2.62 1.38 0.38 0)	0.73
G_{42}	(5.30 4 3.62 1.70 1.38 0)	
$G_{42} \setminus v_1$	(3.62 2.62 1.38 0.38 0)	1
G_{43}	(5.56 4 3 2 1.44 0)	
$G_{43} \setminus v_1$	(4.17 2.31 1 0.52 0)	0.92
G_{44}	$(5.24\ 4\ 3^2\ 0.76\ 0)$	
$G_{44} \setminus v_1$	$(3^2\ 2\ 0^2)$	0.76
G_{45}	(6 5 4 2 1 0)	
$G_{45} \setminus v_1$	$(5\ 3\ 1^2\ 0)$	0
G_{46}	(5.37 5 4.53 2.21 0.89 0)	
$G_{46} \setminus v_1$	$(4^2\ 2\ 0^2)$	0.89
$G_{46} \setminus v_3$	(4.30 3.62 1.38 0.70 0)	0.29
G_{47}	(5.51 5 3.57 3 0.91 0)	
$G_{47} \setminus v_1$	$(4^2\ 2\ 0^2)$	0.91
$G_{47} \setminus v_2$	(4.48 2.69 2 0.83 0)	0.08
G_{48}	(5.24 5 4 3 0.76 0)	
$G_{48} \setminus v_1$	(4.17 3 2.31 0.52 0)	0.24

G_{49}	(6 5.17 3.31 2 1.52 0)	
$G_{49} \setminus v_1$	(5 3 1^2 0)	0.52
G_{50}	(5.30^2 4 1.70^2 0)	
$G_{50} \setminus v_1$	(4.30 3.62 1.38 0.70 0)	1
G_{51}	(5.59 4.86 3.64 2.30 1.61 0)	
$G_{51} \setminus v_1$	(4.30 3.62 1.38 0.70 0)	0.91
$G_{51} \setminus v_2$	(4.48 2.69 2 0.83 0)	0.78
G_{52}	(5.34 5 3.47 3 1.19 0)	
$G_{52} \setminus v_1$	(4.17 3 2.31 0.52 0)	0.67
G_{53}	(5.56 5 3^2 1.44 0)	
$G_{53} \setminus v_1$	(4.48 2.69 2 0.83 0)	0.61
G_{54}	(5.41 4.73 4 2.59 1.27 0)	
$G_{54} \setminus v_1$	(4.17 3 2.31 0.52 0)	0.75
G_{55}	(6 4^2 2^2 0)	
$G_{55} \setminus v_1$	(5 3 1^2 0)	1
G_{56}	(5.30 4.62 4 2.38 1.70 0)	
$G_{56} \setminus v_1$	(3.62^2 1.38^2 0)	0.32
G_{57}	(5.56 4^2 3 1.44 0)	
$G_{57} \setminus v_1$	(4.17 3 2.31 0.52 0)	0.92
G_{58}	(5.68 4.54 3^2 1.79 0)	
$G_{58} \setminus v_1$	(4.48 2.69 2 0.83 0)	0.96
G_{59}	(6 5^2 3 1 0)	
$G_{59} \setminus v_1$	(5 4 2 1 0)	0
G_{60}	(5.51 5^2 3.57 0.91 0)	
$G_{60} \setminus v_1$	(4^3 0^2)	0.91
$G_{60} \setminus v_4$	(4.48 4 2.69 0.83 0)	0.08
G_{61}	(6 5.30 4.62 2.38 1.70 0)	
$G_{61} \setminus v_1$	(5 4 2 1 0)	0.70
G_{62}	(5.56 5^2 3 1.44 0)	
$G_{62} \setminus v_1$	(4.48 4 2.69 0.83 0)	0.61
G_{63}	(6 5.48 3.69 3 1.83 0)	
$G_{63} \setminus v_1$	(5 4 2 1 0)	0.83
G_{64}	(6 5.17 4 3.31 1.52 0)	
$G_{64} \setminus v_1$	(5 3^2 1 0)	0.52
G_{65}	(5.62 5.30 4 3.38 1.70 0)	
$G_{65} \setminus v_1$	(4.48 4 2.69 0.83 0)	0.87
$G_{65} \setminus v_3$	(4.62 3.62 2.38 1.38 0)	0.32
G_{66}	(5.68 5 4.54 3 1.79 0)	
$G_{66} \setminus v_1$	(4.48 4 2.69 0.83 0)	0.96

$G_{66} \setminus v_3$	(4.62 3.62 2.38 1.38 0)	0.41
G_{67}	(6 5 4 3 2 0)	
$G_{67} \setminus v_1$	(5 4 2 1 0)	1
$G_{67} \setminus v_2$	(5 3 2^2 0)	0
G_{68}	(5.73 5 4 3 2.27 0)	
$G_{68} \setminus v_1$	(4.62 3.62 2.38 1.38 0)	0.89
G_{69}	(6 5 3^3 0)	
$G_{69} \setminus v_1$	(5 3 2^2 0)	1
G_{70}	(6 4^3 2 0)	
$G_{70} \setminus v_1$	(5 3^2 1 0)	1
G_{71}	(6 5^3 1 0)	
$G_{71} \setminus v_1$	(5 4^2 1 0)	0
G_{72}	(6^2 5 3 2 0)	
$G_{72} \setminus v_1$	(5^2 2^2 0)	0
G_{73}	(6 5.48 5 3.69 1.83 0)	
$G_{73} \setminus v_1$	(5 4^2 1 0)	0.83
$G_{73} \setminus v_3$	(5 4.41 3 1.59 0)	0.24
G_{74}	(6 5^2 4 2 0)	
$G_{74} \setminus v_1$	(5 4 3 2 0)	0
$G_{74} \setminus v_5$	(5 4^2 1 0)	1
G_{75}	(6 5.62 4.62 3.38 2.38 0)	
$G_{75} \setminus v_1$	(5 4.41 3 1.59 0)	0.79
G_{76}	(6^2 4 3^2 0)	
$G_{76} \setminus v_1$	(5^2 2^2 0)	1
G_{77}	(6 5.41 4^2 2.59 0)	
$G_{77} \setminus v_1$	(5 4 3 2 0)	0.59
$G_{77} \setminus v_4$	(5 4.41 3 1.59 0)	1
G_{78}	(6 5^2 3^2 0)	
$G_{78} \setminus v_1$	(5 4 3 2 0)	1
G_{79}	(6^2 5^2 2 0)	
$G_{79} \setminus v_1$	(5^2 4 2 0)	0
G_{80}	(6^2 5.41 4 2.59 0)	
$G_{80} \setminus v_1$	(5^2 4 2 0)	0.59
G_{81}	(6^2 5 4 3 0)	
$G_{81} \setminus v_1$	(5^2 3^2 0)	0
$G_{81} \setminus v_4$	(5^2 4 2 0)	1
G_{82}	(6^2 4^3 0)	
$G_{82} \setminus v_1$	(5^2 3^2 0)	1
G_{83}	(6^3 5 3 0)	
$G_{83} \setminus v_1$	(5^3 3 0)	0

G_{84}	$(6^3\ 4^2\ 0)$	
$G_{84} \setminus v_1$	$(5^3\ 3\ 0)$	1
G_{85}	$(6^4\ 4\ 0)$	
$G_{85} \setminus v_1$	$(5^4\ 0)$	-1

Tabela B.4: Grafos com 7 vértices

$Grafo$	$Spec$	$\phi(v)$
G_{86}	$(6.05\ 2.48\ 1^3\ 0.47\ 0)$	
$G_{86} \setminus v_1$	$(2\ 0^5)$	0.47
G_{87}	$(6.06\ 3.83\ 1.52\ 1^2\ 0.60\ 0)$	
$G_{87} \setminus v_1$	$(3\ 1\ 0^4)$	0.60
G_{88}	$(6.05\ 3\ 2.48\ 1^2\ 0.47\ 0)$	
$G_{88} \setminus v_1$	$(2^2\ 0^4)$	0.47
G_{89}	$(6.13\ 3.14\ 2\ 1^2\ 0.73\ 0)$	
$G_{89} \setminus v_1$	$(3\ 1\ 0^4)$	0.73
G_{90}	$(6.07\ 4.88\ 2\ 1.34\ 1\ 0.70\ 0)$	
$G_{90} \setminus v_1$	$(4\ 1^2\ 0^3)$	0.70
G_{91}	$(6.06\ 4.34\ 2.77\ 1.19\ 1\ 0.65\ 0)$	
$G_{91} \setminus v_1$	$(3.41\ 2\ 0.59\ 0^3)$	0.65
G_{92}	$(6.13\ 4\ 3.14\ 1^2\ 0.73\ 0)$	
$G_{92} \setminus v_1$	$(3^2\ 0^4)$	0.73
G_{93}	$(6.06\ 3.83\ 3\ 1.52\ 1\ 0.60\ 0)$	
$G_{93} \setminus v_1$	$(3\ 2\ 1\ 0^3)$	0.60
G_{94}	$(6.14\ 3.96\ 2.60\ 1.47\ 1\ 0.83\ 0)$	
$G_{94} \setminus v_1$	$(3.41\ 2\ 0.59\ 0^3)$	0.83
G_{95}	$(6.05\ 4\ 2.48\ 2\ 1\ 0.47\ 0)$	
$G_{95} \setminus v_1$	$(3\ 2\ 1\ 0^3)$	0.47
G_{96}	$(6.27\ 3.86\ 2^2\ 1\ 0.87\ 0)$	
$G_{96} \setminus v_1$	$(4\ 1^2\ 0^3)$	0.87
G_{97}	$(6.13\ 3.14\ 3\ 2\ 1\ 0.73\ 0)$	
$G_{97} \setminus v_1$	$(3\ 2\ 1\ 0^3)$	0.73
G_{98}	$(6.05\ 3^2\ 2.48\ 1\ 0.47\ 0)$	
$G_{98} \setminus v_1$	$(2^3\ 0^3)$	0.47
G_{99}	$(6.07\ 4.88\ 4\ 1.34\ 1\ 0.70\ 0)$	
$G_{99} \setminus v_1$	$(4\ 3\ 1\ 0^3)$	0.70
G_{100}	$(6.19\ 5.79\ 2^2\ 1.21\ 0.81\ 0)$	
$G_{100} \setminus v_1$	$(5\ 1^3\ 0^2)$	0.81

G_{101}	(6.08 5.07 3.26 1.71 1.13 0.76 0)	
$G_{101} \setminus v_1$	(4.17 2.31 1 0.52 0^2)	0.76
G_{102}	(6.05 5.14 3.03 2 1.08 0.69 0)	
$G_{102} \setminus v_1$	(4.17 2.31 1 0.52 0^2)	0.69
G_{103}	(6.14 4.92 3.36 1.74 1 0.83 0)	
$G_{103} \setminus v_1$	(4 3 1 0^3)	0.83
G_{104}	(6.06 4.58 3.49 2.15 1.06 0.67 0)	
$G_{104} \setminus v_1$	(3.62 2.62 1.38 0.38 0)	0.29
G_{105}	(6.27 4 3.86 2 1 0.87 0)	
$G_{105} \setminus v_1$	(4 3 1 0^3)	0.87
G_{106}	(6.16 4.69 3 2.31 1 0.84 0)	
$G_{106} \setminus v_1$	(4 2^2 0^3)	0.84
G_{107}	(6.05 4^2 2.48 1 0.47 0)	
$G_{107} \setminus v_1$	(3^2 2 0^3)	0.47
G_{108}	(6.16 4.93 2.67 2 1.33 0.91 0)	
$G_{108} \setminus v_1$	(4.17 2.31 1 0.52 0^2)	0.91
G_{109}	(6.07 4.88 3 2 1.34 0.70 0)	
$G_{109} \setminus v_1$	(4 2 1^2 0^2)	0.70
G_{110}	(6.05 5 2.48 2^2 0.47 0)	
$G_{110} \setminus v_1$	(4 2 1^2 0^2)	0.47
G_{111}	(6.53 4.52 2^3 0.95 0)	
$G_{111} \setminus v_1$	(5 1^3 0^2)	0.95
G_{112}	(6.14 4.38 3.09 2.31 1.19 0.89 0)	
$G_{112} \setminus v_1$	(3.62 2.62 1.38 0.38 0^2)	0.89
G_{113}	(6.05 4.41 3 2.48 1.59 0.47 0)	
$G_{113} \setminus v_1$	(3.41 2^2 0.59 0^2)	0.47
G_{114}	(6.06 4.34 3 2.77 1.19 0.65 0)	
$G_{114} \setminus v_1$	(3.41 2^2 0.59 0^2)	0.65
G_{115}	(6.06 4 3.83 2 1.52 0.60 0)	
$G_{115} \setminus v_1$	(3^2 1^2 0^2)	0.60
G_{116}	(6.15 4.14 3.62 1.79 1.38 0.92 0)	
$G_{116} \setminus v_1$	(3.62 2.62 1.38 0.38 0^2)	0.92
G_{117}	(6.29 4.24 3.08 2 1.45 0.94 0)	
$G_{117} \setminus v_1$	(4.17 2.31 1 0.52 0^2)	0.94
G_{118}	(6.13 4 3.14 3 1 0.73 0)	
$G_{118} \setminus v_1$	(3^2 2 0^3)	0.73
G_{119}	(6.14 3.96 3 2.60 1.47 0.83 0)	
$G_{119} \setminus v_1$	(3.41 2^2 0.59 0^2)	0.83
G_{120}	(6.13 4 3.14 2^2 0.73 0)	

$G_{120} \setminus v_1$	$(3^2\ 1^2\ 0^2)$	0.73
G_{121}	(6.27 3.86 3 2^2 0.87 0)	
$G_{121} \setminus v_1$	$(4\ 2\ 1^2\ 0^2)$	0.87
G_{122}	(6.19 5.79 4 2 1.21 0.81 0)	
$G_{122} \setminus v_1$	$(5\ 3\ 1^2\ 0^2)$	0.81
G_{123}	(6.08 5.24 4.57 2.22 1.14 0.76 0)	
$G_{123} \setminus v_1$	$(4.30\ 3.62\ 1.38\ 0.70\ 0^2)$	0.76
G_{124}	(6.16 5 4.69 2.31 1 0.84 0)	
$G_{124} \setminus v_1$	$(4^2\ 2\ 0^3)$	0.84
G_{125}	(6.08 5.40 3.59 3 1.15 0.77 0)	
$G_{125} \setminus v_1$	$(4.48\ 2.69\ 2\ 0.83\ 0^2)$	0.77
G_{126}	(6.06 5.14 4 3.03 1.08 0.69 0)	
$G_{126} \setminus v_1$	$(4.17\ 3\ 2.31\ 0.52\ 0^2)$	0.69
G_{127}	(6.27 5 3.86 3 1 0.87 0)	
$G_{127} \setminus v_1$	$(4^2\ 2\ 0^3)$	0.87
G_{128}	(6.23 5.85 3.41 2 1.60 0.92 0)	
$G_{128} \setminus v_1$	$(5\ 3\ 1^2\ 0^2)$	0.92
$G_{128} \setminus v_2$	$(5.09\ 2.43\ 1^2\ 0.49\ 0)$	0.43
G_{129}	(6.13 6 3.14 2^2 0.73 0)	
$G_{129} \setminus v_1$	$(5.09\ 2.43\ 1^2\ 0.49\ 0)$	0.24
G_{130}	(6.08 5.49 3.91 2 1.69 0.83 0)	
$G_{130} \setminus v_1$	$(4.56\ 3\ 1^2\ 0.44\ 0)$	0.39
G_{131}	(6.15 5.30 4.14 1.79 1.70 0.92 0)	
$G_{131} \setminus v_1$	$(4.30\ 3.62\ 1.38\ 0.70\ 0^2)$	0.92
G_{132}	(6.29 4.94 3.89 2.32 1.62 0.94 0)	
$G_{132} \setminus v_1$	$(4.30\ 3.62\ 1.38\ 0.70\ 0^2)$	0.94
G_{133}	(6.06 5.28 3.51 2.82 1.62 0.71 0)	
$G_{133} \setminus v_1$	(4.30 2.62 2 0.70 0.38 0)	0.33
G_{134}	(6.15 5.10 3.51 3.08 1.27 0.90 0)	
$G_{134} \setminus v_1$	$(4.17\ 3\ 2.31\ 0.52\ 0^2)$	0.90
G_{135}	(6.08 5.12 3.96 2.43 1.61 0.80 0)	
$G_{135} \setminus v_1$	(4.21 3 1.46 1 0.32 0)	0.48
G_{136}	(6.19 5.21 3.65 2.37 1.66 0.92 0)	
$G_{136} \setminus v_1$	$(4.48\ 2.69\ 2\ 0.83\ 0^2)$	0.92
G_{137}	(6.05 5 4 2.48 2 0.47 0)	
$G_{137} \setminus v_1$	$(4\ 3\ 2\ 1\ 0^2)$	0.47
G_{138}	(6.16 4.93 4 2.67 1.33 0.91 0)	
$G_{138} \setminus v_1$	$(4.17\ 3\ 2.31\ 0.52\ 0^2)$	0.91
G_{139}	(6.08 5.21 3.62 2.86 1.38 0.85 0)	
$G_{139} \setminus v_1$	(4.30 2.62 2 0.70 0.38 0)	0.47

G_{140}	(6.06 5.21 3.87 2.19 2 0.68 0)	
$G_{140} \setminus v_1$	(4.21 3 1.46 1 0.32 0)	0.36
G_{141}	(6.17 5.31 3.09 3 1.51 0.92 0)	
$G_{141} \setminus v_1$	(4.48 2.69 2 0.83 0^2)	0.92
G_{142}	(6.07 4.88 4 3 1.34 0.70 0)	
$G_{142} \setminus v_1$	(4 3 2 1 0^2)	0.70
G_{143}	(6.53 4.52 4 2^2 0.95 0)	
$G_{143} \setminus v_1$	(5 3 1^2 0^2)	0.95
G_{144}	(6.06 4.71 3.92 2.85 1.79 0.68 0)	
$G_{144} \setminus v_1$	(3.73 3 2 1 0.27 0)	0.41
G_{145}	(6.32 4.81 3.13 3 1.79 0.94 0)	
$G_{145} \setminus v_1$	(4.48 2.69 2 0.83 0^2)	0.94
G_{146}	(6.15 4.62 4.14 2.38 1.79 0.92 0)	
$G_{146} \setminus v_1$	(3.62^2 1.38^2 0^2)	0.92
G_{147}	(6.29 4.24 4 3.08 1.45 0.94 0)	
$G_{147} \setminus v_1$	(4.17 3 2.31 0.52 0^2)	0.94
G_{148}	(6.06 4^2 3.83 1.52 0.60 0)	
$G_{148} \setminus v_1$	(3^3 1 0^2)	0.60
G_{149}	(6.05 5 3^2 2.48 0.47 0)	
$G_{149} \setminus v_1$	(4 2^3 0^2)	0.47
G_{150}	(6.30 5.79 2.70 2^2 1.21 0)	
$G_{150} \setminus v_1$	(5.09 2.43 1^2 0.49 0)	0.72
G_{151}	(7 5 2^4 0)	
$G_{151} \setminus v_1$	(6 1^4 0)	1
G_{152}	(6.17 5.10 3.32 2.62 1.68 1.11 0)	
$G_{152} \setminus v_1$	(4.30 2.62 2 0.70 0.38 0)	0.73
G_{153}	(6.15 5.15 3.34 2.35 2 1.01 0)	
$G_{153} \setminus v_1$	(4.21 3 1.46 1 0.32 0)	0.69
G_{154}	(6.14 4.92 3.36 3 1.74 0.83 0)	
$G_{154} \setminus v_1$	(4 3 2 1 0^2)	0.83
G_{155}	(6.17 4.94 3.84 2 1.75 1.30 0)	
$G_{155} \setminus v_1$	(4.21 3 1.46 1 0.32 0)	0.98
G_{156}	(6.32 5 3.36 2^2 1.32 0)	
$G_{156} \setminus v_1$	(4.56 3 1^2 0.44 0)	0.88
G_{157}	(6.56 4.68 3.32 2^2 1.44 0)	
$G_{157} \setminus v_1$	(5.09 2.43 1^2 0.49 0)	0.95
G_{158}	(6.14 4.60 3.61 2.68 2 0.98 0)	
$G_{158} \setminus v_1$	(3.73 3 2 1 0.27 0)	0.71
G_{159}	(6.15 4.41 3.86 2.80 1.59 1.19 0)	

$G_{159} \setminus v_1$	(3.73 3 2 1 0.27 0)	0.92
G_{160}	(6.29 4.49 3.60 2.43 2 1.19 0)	
$G_{160} \setminus v_1$	(4.21 3 1.46 1 0.32 0)	0.87
G_{161}	(6.30 4.41 3.62 2.70 1.59 1.38 0)	
$G_{161} \setminus v_1$	(4.30 2.62 2 0.70 0.38 0)	1
G_{162}	(6.16 4.69 3^2 2.31 0.84 0)	
$G_{162} \setminus v_1$	(4 2^3 0^2)	0.84
G_{163}	(6.27 4 3.86 3 2 0.87 0)	
$G_{163} \setminus v_1$	(4 3 2 1 0^2)	0.87
G_{164}	(6.13 4^2 3.14 2 0.73 0)	
$G_{164} \setminus v_1$	(3^3 1 0^2)	0.73
G_{165}	(6.13 4^2 3.14 2 0.73 0)	
$G_{165} \setminus v_1$	(3^3 1 0^2)	0.73
G_{166}	(6.19 5.79 5 3 1.21 0.81 0)	
$G_{166} \setminus v_1$	(5 4 2 1 0^2)	0.81
G_{167}	(6.08 5.40 5 3.59 1.15 0.77 0)	
$G_{167} \setminus v_1$	(4.48 4 2.69 0.83 0^2)	0.77
G_{168}	(6.27 5^2 3.86 1 0.87 0)	
$G_{168} \setminus v_1$	(4^3 0^3)	0.87
G_{169}	(7 6 4 2^2 1 0)	
$G_{169} \setminus v_1$	(6 3 1^3 0)	0
G_{170}	(6.24 5.86 4.78 2.43 1.76 0.92 0)	
$G_{170} \setminus v_1$	(5.11 3.79 1.47 1 0.63 0)	0.29
$G_{170} \setminus v_2$	(5 4 2 1 0^2)	0.92
G_{171}	(6.16 6 4.69 2.31 2 0.84 0)	
$G_{171} \setminus v_1$	(5.11 3.79 1.47 1 0.63 0)	0.21
G_{172}	(6.17 5.31 5 3.09 1.51 0.92 0)	
$G_{172} \setminus v_1$	(4.48 4 2.69 0.83 0^2)	0.92
G_{173}	(6.08 5.30 5.21 2.86 1.70 0.85 0)	
$G_{173} \setminus v_1$	(4.30^2 2 0.70^2 0)	0.15
G_{174}	(6.31 5.92 3.97 3 1.86 0.94 0)	
$G_{174} \setminus v_1$	(5 4 2 1 0^2)	0.94
$G_{174} \setminus v_2$	(5.24 3 2 1 0.76 0)	0.18
G_{175}	(6.23 5.85 4 3.41 1.60 0.92 0)	
$G_{175} \setminus v_1$	(5 3^2 1 0^2)	0.92
$G_{175} \setminus v_2$	(5.09 3 2.43 1 0.49 0)	0.43
G_{176}	(6.13 6 4 3.14 2 0.73 0)	
$G_{176} \setminus v_1$	(5.09 3 2.43 1 0.49 0)	0.24
G_{177}	(6.29 5.38 4.18 3.49 1.72 0.94 0)	
$G_{177} \setminus v_1$	(4.48 4 2.69 0.83 0^2)	0.94

G_{178}	(6.10 5.73 3.92 3.49 1.90 0.85 0)	
$G_{178} \setminus v_1$	(4.81 3 2.53 1 0.66 0)	0.19
G_{179}	(6.20 5.39 4.57 3.04 1.88 0.92 0)	
$G_{179} \setminus v_1$	(4.62 3.62 2.38 1.38 0^2)	0.92
G_{180}	(6.06 5.38 4.68 3.11 2.06 0.71 0)	
$G_{180} \setminus v_1$	(4.39 3.70 2.36 1.14 0.41 0)	0.30
G_{181}	(6.53 5 4.52 3 2 0.95 0)	
$G_{181} \setminus v_1$	(5 4 2 1 0^2)	0.95
G_{182}	(6.08 5.49 4 3.91 1.69 0.83 0)	
$G_{182} \setminus v_1$	(4.56 3^2 1 0.44 0)	0.39
G_{183}	(6.32 5 4.81 3.13 1.79 0.94 0)	
$G_{183} \setminus v_1$	(4.48 4 2.69 0.83 0^2)	0.94
G_{184}	(6.15 5.62 4.14 3.38 1.79 0.92 0)	
$G_{184} \setminus v_1$	(4.62 3.62 2.38 1.38 0^2)	0.92
G_{185}	(6.05 5^2 3 2.48 0.47 0)	
$G_{185} \setminus v_1$	(4^2 2^2 0^2)	0.47
G_{186}	(6.32 5.21 4.15 3.08 2.29 0.94 0)	
$G_{186} \setminus v_1$	(4.62 3.62 2.38 1.38 0^2)	0.94
G_{187}	(6.08 5.21 4.62 2.86 2.38 0.85 0)	
$G_{187} \setminus v_1$	(4.30 3.62 2 1.38 0.70 0)	0.15
G_{188}	(6.06 5.21 4 3.87 2.19 0.68 0)	
$G_{188} \setminus v_1$	(4.21 3^2 1.46 0.32 0)	0.36
G_{189}	(6.40 5.48 3.17 3^2 0.94 0)	
$G_{189} \setminus v_1$	(5 3 2^2 0^2)	0.94
G_{190}	(6.53 4.52 4^2 2 0.95 0)	
$G_{190} \setminus v_1$	(5 3^2 1 0^2)	0.95
G_{191}	(7 6.09 3.43 2^2 1.49 0)	
$G_{191} \setminus v_1$	(6 3 1^3 0)	0.49
G_{192}	(6.24 6 4.41 2 1.76 1.59 0)	
$G_{192} \setminus v_1$	(5.11 3.79 1.47 1 0.63 0)	0.96
G_{193}	(6.25 5.91 3.52 3.29 1.80 1.23 0)	
$G_{193} \setminus v_1$	(5.09 3 2.43 1 0.49 0)	0.74
G_{194}	(6.38 5.85 3.89 2.38 2 1.50 0)	
$G_{194} \setminus v_1$	(5.11 3.79 1.47 1 0.63 0)	0.87
$G_{194} \setminus v_2$	(5.24 3 2 1 0.76 0)	0.74
G_{195}	(6.30 5.79 4 2.70 2 1.21 0)	
$G_{195} \setminus v_1$	(5.09 3 2.43 1 0.49 0)	0.72
G_{196}	(6.32 6 3.36 3 2 1.32 0)	
$G_{196} \setminus v_1$	(5.24 3 2 1 0.76 0)	0.56

G_{197}	(6.17 5.25 4.59 2.73 2.13 1.12 0)	
$G_{197} \setminus v_1$	(4.39 3.70 2.36 1.14 0.41 0)	0.71
G_{198}	(6.23 5.59 3.95 3.06 1.69 1.49 0)	
$G_{198} \setminus v_1$	(4.81 3 2.53 1 0.66 0)	0.83
G_{199}	(6.15 5.39 4.31 3.14 1.75 1.26 0)	
$G_{199} \setminus v_1$	(4.39 3.70 2.36 1.14 0.41 0)	0.85
G_{200}	(6.56 5 4.41 2.44 2 1.59 0)	
$G_{200} \setminus v_1$	(5.11 3.79 1.47 1 0.63 0)	0.96
G_{201}	(6.15 5.51 4 3.26 2 1.08 0)	
$G_{201} \setminus v_1$	(4.56 3^2 1 0.44 0)	0.64
G_{202}	(6.24 5.41 4.41 2.59 1.76 1.59 0)	
$G_{202} \setminus v_1$	(4.73 3.41 2 1.27 0.59 0)	1
G_{203}	(6.30 5.30 4.41 2.70 1.70 1.59 0)	
$G_{203} \setminus v_1$	(4.30^2 2 0.70^2 0)	0.89
G_{204}	(6.16 5 4.69 3 2.31 0.84 0)	
$G_{204} \setminus v_1$	(4^2 2^2 0^2)	0.84
G_{205}	(6.37 5.26 3.68 3.13 2 1.56 0)	
$G_{205} \setminus v_1$	(4.81 3 2.53 1 0.66 0)	0.90
G_{206}	(6.31 4.95 4.21 3.22 1.90 1.41 0)	
$G_{206} \setminus v_1$	(4.39 3.70 2.36 1.14 0.41 0)	1
G_{207}	(6.18 5.13 4.41 2.87 1.82 1.59 0)	
$G_{207} \setminus v_1$	(4.30 3.62 2 1.38 0.70 0)	0.89
G_{208}	(6.29 5.12 3.89 3.32 2.12 1.26 0)	
$G_{208} \setminus v_1$	(4.39 3.70 2.36 1.14 0.41 0)	0.85
G_{209}	(6.15 5.29 4.19 2.83 2.19 1.35 0)	
$G_{209} \setminus v_1$	(4.30 3.62 2 1.38 0.70 0)	0.65
G_{210}	(6.15 5.15 4 3.34 2.35 1.01 0)	
$G_{210} \setminus v_1$	(4.21 3^2 1.46 0.32 0)	0.69
G_{211}	(6.20 5.31 3.92 3 2.34 1.23 0)	
$G_{211} \setminus v_1$	(4.56 3 2^2 0.44 0)	0.79
G_{212}	(6.34 5.34 3.65 3 2.23 1.45 0)	
$G_{212} \setminus v_1$	(4.73 3.41 2 1.27 0.59 0)	0.86
G_{213}	(6.56 4.68 4 3.32 2 1.44 0)	
$G_{213} \setminus v_1$	(5.09 3 2.43 1 0.49 0)	0.95
G_{214}	(6.15 5.15 4 3.34 2.35 1.01 0)	
$G_{214} \setminus v_1$	(4.21 3^2 1.46 0.32 0)	0.69
G_{215}	(6.17 4.94 4 3.84 1.75 1.30 0)	
$G_{215} \setminus v_1$	(4.21 3^2 1.46 0.32 0)	0.98
G_{216}	(6.18 5.41 3.68 3 2.59 1.14 0)	

$G_{216} \setminus v_1$	(4.56 3 2^2 0.44 0)	0.70
G_{217}	(6.18 5.41 3.68 3 2.59 1.14 0)	
$G_{217} \setminus v_1$	(4.56 3 2^2 0.44 0)	0.70
G_{218}	(6.27 5 3.86 3^2 0.87 0)	
$G_{218} \setminus v_1$	(4^2 2^2 0^2)	0.87
G_{219}	(6.60 5 3.66 3 2 1.74 0)	
$G_{219} \setminus v_1$	(5.24 3 2 1 0.76 0)	0.98
G_{220}	(6.32 5 4 3.36 2 1.32 0)	
$G_{220} \setminus v_1$	(4.56 3^2 1 0.44 0)	0.88
G_{221}	(6.32 5 4 3.36 2 1.32 0)	
$G_{221} \setminus v_1$	(4.56 3^2 1 0.44 0)	0.88
G_{222}	(6.30 4.62 4.41 2.70 2.38 1.59 0)	
$G_{222} \setminus v_1$	(4.30 3.62 2 1.38 0.70 0)	0.89
G_{223}	(6.29 4.49 4 3.60 2.43 1.19 0)	
$G_{223} \setminus v_1$	(4.21 3^2 1.46 0.32 0)	0.87
G_{224}	(6.33 4.85 3.87 3 2.53 1.42 0)	
$G_{224} \setminus v_1$	(4.56 3 2^2 0.44 0)	0.98
G_{225}	(6.15 4.83 4 3.56 2 1.45 0)	
$G_{225} \setminus v_1$	(4 3^2 1^2 0)	0.45
G_{226}	(6.13 4^3 3.14 0.73 0)	
$G_{226} \setminus v_1$	(3^4 0^2)	0.73
G_{227}	(6.19 5.79 5^2 1.21 0.81 0)	
$G_{227} \setminus v_1$	(5 4^2 1 0^2)	0.81
G_{228}	(7 6 5 3 2 1 0)	
$G_{228} \setminus v_1$	(6 4 2 1^2 0)	0
G_{229}	(6.30 6 5.66 3 2.12 0.93 0)	
$G_{229} \setminus v_1$	(5^2 2^2 0^2)	0.93
$G_{229} \setminus v_3$	(5.24 4.73 2 1.27 0.76 0)	0.17
G_{230}	(6.24 5.87 5.29 3.74 1.93 0.92 0)	
$G_{230} \setminus v_1$	(5 4.41 3 1.59 0^2)	0.92
$G_{230} \setminus v_2$	(5.11 4.30 2.75 1.14 0.70 0)	0.22
G_{231}	(6.17 6 5.26 3.64 2.08 0.86 0)	
$G_{231} \setminus v_1$	(5.11 4.30 2.75 1.14 0.70 0)	0.16
G_{232}	(6.31 5.92 5 3.97 1.86 0.94 0)	
$G_{232} \setminus v_1$	(5 4^2 1 0^2)	0.94
$G_{232} \setminus v_2$	(5.24 4 3 1 0.76 0)	0.18
G_{233}	(6.09 5.67 5 4.33 2.07 0.84 0)	
$G_{233} \setminus v_1$	(4.73 4 3.41 1.27 0.59 0)	0.25
G_{234}	(6.30 5.66 5 4 2.12 0.93 0)	
$G_{234} \setminus v_1$	(5 4 3 2 0^2)	0.93

G_{235}	$(6.53\ 5^2\ 4.52\ 2\ 0.95\ 0)$	
$G_{235} \setminus v_1$	$(5\ 4^2\ 1\ 0^2)$	0.95
G_{236}	$(6.05\ 5^3\ 2.48\ 0.47\ 0)$	
$G_{236} \setminus v_1$	$(4^3\ 2\ 0^2)$	0.47
G_{237}	$(7\ 6\ 4^2\ 2\ 1\ 0)$	
$G_{237} \setminus v_1$	$(6\ 3^2\ 1^2\ 0)$	0
G_{238}	$(6.33\ 5.94\ 4.85\ 3.51\ 2.42\ 0.94\ 0)$	
$G_{238} \setminus v_1$	$(5.27\ 3.86\ 2.53\ 1.45\ 0.88\ 0)$	0.06
$G_{238} \setminus v_2$	$(5\ 4.41\ 3\ 1.59\ 0^2)$	0.94
G_{239}	$(6.13\ 6\ 5\ 3.14\ 3\ 0.73\ 0)$	
$G_{239} \setminus v_1$	$(5.09\ 4\ 2.43\ 2\ 0.49\ 0)$	0.24
G_{240}	$(6.16\ 6\ 4.69\ 4\ 2.31\ 0.84\ 0)$	
$G_{240} \setminus v_1$	$(5.11\ 3.79\ 3\ 1.47\ 0.63\ 0)$	0.21
G_{241}	$(6.53\ 6\ 4.52\ 3^2\ 0.95\ 0)$	
$G_{241} \setminus v_1$	$(5.51\ 3.57\ 2^2\ 0.91\ 0)$	0.04
$G_{241} \setminus v_2$	$(5^2\ 2^2\ 0^2)$	0.95
G_{242}	$(6.11\ 5.81\ 4.69\ 3.91\ 2.62\ 0.86\ 0)$	
$G_{242} \setminus v_1$	$(4.89\ 3.70\ 3\ 1.68\ 0.72\ 0)$	0.06
G_{243}	$(6.08\ 5.62\ 5.21\ 3.38\ 2.86\ 0.85\ 0)$	
$G_{243} \setminus v_1$	$(4.62\ 4.30\ 2.38\ 2\ 0.70\ 0)$	0.15
G_{244}	$(6.06\ 5.55\ 5\ 3.92\ 2.76\ 0.71\ 0)$	
$G_{244} \setminus v_1$	$(4.56\ 4\ 3\ 2\ 0.44\ 0)$	0.27
G_{245}	$(6.40\ 5.48\ 5\ 3.17\ 3\ 0.94\ 0)$	
$G_{245} \setminus v_1$	$(5\ 4\ 3\ 2\ 0^2)$	0.94
G_{246}	$(6.53\ 5.41\ 4.52\ 4\ 2.59\ 0.95\ 0)$	
$G_{246} \setminus v_1$	$(5\ 4.41\ 3\ 1.59\ 0^2)$	0.95
G_{247}	$(6.35\ 5.81\ 4.23\ 4\ 2.66\ 0.94\ 0)$	
$G_{247} \setminus v_1$	$(5\ 4\ 3\ 2\ 0^2)$	0.94
G_{248}	$(6.27\ 6\ 4\ 3.86\ 3\ 0.87\ 0)$	
$G_{248} \setminus v_1$	$(5.24\ 3^2\ 2\ 0.76\ 0)$	0.11
G_{249}	$(6.40\ 5.48\ 5\ 3.17\ 3\ 0.94\ 0)$	
$G_{249} \setminus v_1$	$(5\ 4\ 3\ 2\ 0^2)$	0.94
G_{250}	$(7\ 6.11\ 4.79\ 2.47\ 2\ 1.63\ 0)$	
$G_{250} \setminus v_1$	$(6\ 4\ 2\ 1^2\ 0)$	0.63
G_{251}	$(6.24^2\ 5.41\ 2.59\ 1.76^2\ 0)$	
$G_{251} \setminus v_1$	$(5.24\ 4.73\ 2\ 1.27\ 0.76\ 0)$	1
G_{252}	$(6.29\ 5.91\ 5.22\ 3.29\ 1.80\ 1.49\ 0)$	
$G_{252} \setminus v_1$	$(5.11\ 4.30\ 2.75\ 1.14\ 0.70\ 0)$	0.79
G_{253}	$(6.32\ 6\ 5\ 3.36\ 2\ 1.32\ 0)$	

$G_{253} \setminus v_1$	(5.24 4 3 1 0.76 0)	0.56
G_{254}	(7 6.09 4 3.43 2 1.49 0)	
$G_{254} \setminus v_1$	(6 3^2 1^2 0)	0.49
G_{255}	(7 6.24 4 3 2 1.76 0)	
$G_{255} \setminus v_1$	(6 4 2 1^2 0)	0.76
G_{256}	(6.26 5.93 4.78 3.42 2.35 1.26 0)	
$G_{256} \setminus v_1$	(5.11 3.79 3 1.47 0.63 0)	0.63
$G_{256} \setminus v_2$	(5.09 4 2.43 2 0.49 0)	0.77
G_{257}	(6.31 6.10 4.51 3.49 1.90 1.69 0)	
$G_{257} \setminus v_1$	(5.11 4.30 2.75 1.14 0.70 0)	0.99
$G_{257} \setminus v_2$	(5.27 3.86 2.53 1.45 0.88 0)	0.81
G_{258}	(6.40 5.87 4.86 3.14 2.13 1.60 0)	
$G_{258} \setminus v_1$	(5.27 3.86 2.53 1.45 0.88 0)	0.72
$G_{258} \setminus v_2$	(5.11 4.30 2.75 1.14 0.70 0)	0.90
G_{259}	(6.58 5.93 4.55 3 2.23 1.72 0)	
$G_{259} \setminus v_1$	(5.24 4.73 2 1.27 0.76 0)	0.96
$G_{259} \setminus v_2$	(5.51 3.57 2^2 0.91 0)	0.81
G_{260}	(6.36 6 4.71 3.16 2.29 1.49 0)	
$G_{260} \setminus v_1$	(5.27 3.86 2.53 1.45 0.88 0)	0.61
G_{261}	(6.24 6 4.41 4 1.76 1.59 0)	
$G_{261} \setminus v_1$	(5.11 3.79 3 1.47 0.63 0)	0.93
G_{262}	(6.41 6 4.41 3.59 2 1.59 0)	
$G_{262} \setminus v_1$	(5.24 4 3 1 0.76 0)	0.83
G_{263}	(6.30 5.79 5 3 2.70 1.21 0)	
$G_{263} \setminus v_1$	(5.09 4 2.43 2 0.49 0)	0.72
G_{264}	(7 5^2 3 2^2 0)	
$G_{264} \setminus v_1$	(6 4 2 1^2 0)	1
G_{265}	(6.26 5.61 4.65 3.91 1.88 1.68 0)	
$G_{265} \setminus v_1$	(4.89 3.70 3 1.68 0.72 0)	0.96
G_{266}	(6.15 5.89 4.38 3.85 2.37 1.36 0)	
$G_{266} \setminus v_1$	(4.89 3.70 3 1.68 0.72 0)	0.64
G_{267}	(6.20 5.41 5.25 3.17 2.59 1.37 0)	
$G_{267} \setminus v_1$	(4.62 4.30 2.38 2 0.70 0)	0.67
G_{268}	(6.56 5.38 4.60 3.68 2.13 1.65 0)	
$G_{268} \setminus v_1$	(5.11 4.30 2.75 1.14 0.70 0)	0.95
G_{269}	(6.38 5.85 4 3.89 2.38 1.50 0)	
$G_{269} \setminus v_1$	(5.24 3^2 2 0.76 0)	0.74
G_{270}	(6.18 5.41 5 3.68 2.59 1.14 0)	
$G_{270} \setminus v_1$	(4.56 4 3 2 0.44 0)	0.70
G_{271}	(6.32 5.58 4.41 4.13 1.97 1.59 0)	

$G_{271} \setminus v_1$	(4.73 4 3.41 1.27 0.59 0)	1
G_{272}	(6.18 5.41 5 3.68 2.59 1.14 0)	
$G_{272} \setminus v_1$	(4.56 4 3 2 0.44 0)	0.70
G_{273}	(6.34 5.34 5 3.65 2.23 1.45 0)	
$G_{273} \setminus v_1$	(4.73 4 3.41 1.27 0.59 0)	0.86
G_{274}	(6.60 5^2 3.66 2 1.74 0)	
$G_{274} \setminus v_1$	(5.24 4 3 1 0.76 0)	0.98
G_{275}	(6.27 5^2 3.86 3 0.87 0)	
$G_{275} \setminus v_1$	(4^3 2 0^2)	0.87
G_{276}	(6.38 5.85 4 3.89 2.38 1.50 0)	
$G_{276} \setminus v_1$	(5.11 3.79 3 1.47 0.63 0)	0.87
$G_{276} \setminus v_2$	(5.24 3^2 2 0.76 0)	0.74
G_{277}	(6.32 6 4 3.36 3 1.32 0)	
$G_{277} \setminus v_1$	(5.09 4 2.43 2 0.49 0)	0.83
$G_{277} \setminus v_2$	(5.24 3^2 2 0.76 0)	0.56
G_{278}	(6.46 5.91 4.24 3.29 2.30 1.80 0)	
$G_{278} \setminus v_1$	(5.27 3.86 2.53 1.45 0.88 0)	0.92
G_{279}	(6.32 6 4 3.36 3 1.32 0)	
$G_{279} \setminus v_1$	(5.24 3^2 2 0.76 0)	0.56
G_{280}	(6.60 6 3.66 3^2 1.74 0)	
$G_{280} \setminus v_1$	(5.51 3.57 2^2 0.91 0)	0.83
G_{281}	(7 5 4^2 2^2 0)	
$G_{281} \setminus v_1$	(6 3^2 1^2 0)	1
G_{282}	(6.29 5.45 4.32 3.65 3 1.28 0)	
$G_{282} \setminus v_1$	(4.56 4 3 2 0.44 0)	0.84
G_{283}	(6.61 5.26 4.45 3.39 2.43 1.86 0)	
$G_{283} \setminus v_1$	(5.27 3.86 2.53 1.45 0.88 0)	0.98
G_{284}	(6.38 5.41 4.60 3.40 2.59 1.62 0)	
$G_{284} \setminus v_1$	(4.89 3.70 3 1.68 0.72 0)	0.90
G_{285}	(6.35 5.59 4.21 3.61 2.68 1.55 0)	
$G_{285} \setminus v_1$	(4.89 3.70 3 1.68 0.72 0)	0.83
G_{286}	(6.33 5.41 4.74 3.26 2.59 1.67 0)	
$G_{286} \setminus v_1$	(4.62 4.30 2.38 2 0.70 0)	0.97
G_{287}	(6.38 5.41 4.60 3.40 2.59 1.62 0)	
$G_{287} \setminus v_1$	(4.89 3.70 3 1.68 0.72 0)	0.90
G_{288}	(6.18 5.34 4.76 3.90 2.19 1.64 0)	
$G_{288} \setminus v_1$	(4.41 4 3 1.59 1 0)	0.64
G_{289}	(6.26 5.77 4 3.67 2.78 1.52 0)	
$G_{289} \setminus v_1$	(5 3^2 2 1 0)	0.52
G_{290}	(6.33 5 4.85 3.87 2.53 1.42 0)	

$G_{290} \setminus v_1$	(4.56 4 3 2 0.44 0)	0.98
G_{291}	(6.15 5.41 4.83 3.56 2.59 1.45 0)	
$G_{291} \setminus v_1$	(4.41 4 3 1.59 1 0)	0.45
G_{292}	(6.56 5 4.68 3.32 3 1.44 0)	
$G_{292} \setminus v_1$	(5.09 4 2.43 2 0.49 0)	0.95
G_{293}	(6.56 5 4.41 4 2.44 1.59 0)	
$G_{293} \setminus v_1$	(5.11 3.79 3 1.47 0.63 0)	0.96
G_{294}	(6.15 5.41 4.83 3.56 2.59 1.45 0)	
$G_{294} \setminus v_1$	(4.41 4 3 1.59 1 0)	0.45
G_{295}	(6.30 5.62 4.41 3.38 2.70 1.59 0)	
$G_{295} \setminus v_1$	(4.62 4.30 2.38 2 0.70 0)	0.89
G_{296}	(6.18 5.70 4.67 3 2.79 1.66 0)	
$G_{296} \setminus v_1$	(4.73 4 2^2 1.27 0)	0.39
G_{297}	(6.24 5.41 5 3 2.59 1.76 0)	
$G_{297} \setminus v_1$	(4.73 4 2^2 1.27 0)	0.49
G_{298}	(6.49 5.66 4 3.16 3 1.69 0)	
$G_{298} \setminus v_1$	(5.24 3^2 2 0.76 0)	0.93
G_{299}	(6.15 5.51 4^2 3.26 1.08 0)	
$G_{299} \setminus v_1$	(4.56 3^3 0.44 0)	0.64
G_{300}	(6.49 5.66 4 3.16 3 1.69 0)	
$G_{300} \setminus v_1$	(5.24 3^2 2 0.76 0)	0.93
G_{301}	(6.70 5.51 3.88 3^2 1.90 0)	
$G_{301} \setminus v_1$	(5.51 3.57 2^2 0.91 0)	0.99
G_{302}	(6.31 5.15 4.48 3.77 2.47 1.82 0)	
$G_{302} \setminus v_1$	(4.41 4 3 1.59 1 0)	0.82
G_{303}	(6.41 5.41 4 3.59 2.59 2 0)	
$G_{303} \setminus v_1$	(5 3^2 2 1 0)	1
G_{304}	(6.33 4.85 3.87 3 2.53 1.42 0)	
$G_{304} \setminus v_1$	(4.56 3 2^2 0.44 0)	0.98
G_{305}	(6.60 5 4 3.66 3 1.74 0)	
$G_{305} \setminus v_1$	(5.24 3^2 2 0.76 0)	0.98
G_{306}	(6.32 5 4^2 3.36 1.32 0)	
$G_{306} \setminus v_1$	(4.56 3^3 0.44 0)	0.88
G_{307}	(6.30 6 5.66 5 2.12 0.93 0)	
$G_{307} \setminus v_1$	(5^2 4 2 0^2)	0.93
$G_{307} \setminus v_3$	(5.24 4.73 4 1.27 0.76 0)	0.17
G_{308}	(7 6 5^2 2 1 0)	
$G_{308} \setminus v_1$	(6 4^2 1^2 0)	0
G_{309}	(7 6^2 3^2 1 0)	
$G_{309} \setminus v_1$	(6 5 2^2 1 0)	0

G_{310}	(7 6 5.41 4 2.59 1 0)	
$G_{310} \setminus v_1$	(6 4.41 3 1.59 1 0)	0
G_{311}	(6.35 6 5.81 4.23 2.66 0.94 0)	
$G_{311} \setminus v_1$	(5^2 4 2 0^2)	0.94
$G_{311} \setminus v_3$	(5.30 4.86 3.25 1.70 0.89 0)	0.05
G_{312}	(6.27 6^2 3.86 3 0.87 0)	
$G_{312} \setminus v_1$	(5.24 5 3 2 0.76 0)	0.11
G_{313}	(6.53 6 5 4.52 3 0.95 0)	
$G_{313} \setminus v_1$	(5.51 4 3.57 2 0.91 0)	0.04
$G_{313} \setminus v_2$	(5^2 4 2 0^2)	0.95
G_{314}	(6.40 6 5.48 4 3.17 0.94 0)	
$G_{314} \setminus v_1$	(5^2 3^2 0^2)	0.94
$G_{314} \setminus v_2$	(5.37 4.53 3 2.21 0.89 0)	0.05
G_{315}	(6.17 6 5.53 4.40 3.04 0.86 0)	
$G_{315} \setminus v_1$	(5.12 4.55 3.47 2.14 0.73 0)	0.13
G_{316}	(6.13 6 5^2 3.14 0.73 0)	
$G_{316} \setminus v_1$	(5.09 4^2 2.43 0.49 0)	0.24
G_{317}	(6.27 6 5 4 3.86 0.87 0)	
$G_{317} \setminus v_1$	(5.24 4 3^2 0.76 0)	0.11
G_{318}	(6.53 6 4.52 4^2 0.95 0)	
$G_{318} \setminus v_1$	(5^2 3^2 0^2)	0.95
G_{319}	(7 6.24 5.73 3 2.27 1.76 0)	
$G_{319} \setminus v_1$	(6 5 2^2 1 0)	0.76
G_{320}	(7 6.11 5.30 3.75 2.14 1.70 0)	
$G_{320} \setminus v_1$	(6 4.41 3 1.59 1 0)	0.70
G_{321}	(7 6.24 5 4 2 1.76 0)	
$G_{321} \setminus v_1$	(6 4^2 1^2 0)	0.76
G_{322}	(6.32 6.24 5.58 4.13 1.97 1.76 0)	
$G_{322} \setminus v_1$	(5.24 4.73 4 1.27 0.76 0)	1
$G_{322} \setminus v_3$	(5.30 4.86 3.25 1.70 0.89 0)	0.87
G_{323}	(6.38 6 5.85 3.89 2.38 1.50 0)	
$G_{323} \setminus v_1$	(5.24 5 3 2 0.76 0)	0.74
$G_{323} \setminus v_2$	(5.30 4.86 3.25 1.70 0.89 0)	0.61
G_{324}	(6.26 6 5.41 4.34 2.59 1.40 0)	
$G_{324} \setminus v_1$	(5.12 4.55 3.47 2.14 0.73 0)	0.67
G_{325}	(6.58 5.93 5 4.55 2.23 1.72 0)	
$G_{325} \setminus v_1$	(5.24 4.73 4 1.27 0.76 0)	0.96
$G_{325} \setminus v_2$	(5.51 4 3.57 2 0.91 0)	0.81
G_{326}	(6.45 6 5.41 4 2.59 1.55 0)	
$G_{326} \setminus v_1$	(5.37 4.53 3 2.21 0.89 0)	0.66

G_{327}	(6.30 5.79 5^2 2.70 1.21 0)	
$G_{327} \setminus v_1$	(5.09 4^2 2.43 0.49 0)	0.72
G_{328}	(7 5^3 2^2 0)	
$G_{328} \setminus v_1$	(6 4^2 1^2 0)	1
G_{329}	(7 6.27 4.86 3.53 2.45 1.88 0)	
$G_{329} \setminus v_1$	(6 4.41 3 1.59 1 0)	0.88
G_{330}	(7 6.11 4.79 4 2.47 1.63 0)	
$G_{330} \setminus v_1$	(6 4 3 2 1 0)	0.63
G_{331}	(7 6.09 5 3.43 3 1.49 0)	
$G_{331} \setminus v_1$	(6 4 3 2 1 0)	0.49
G_{332}	(7 6.51 4.57 3^2 1.91 0)	
$G_{332} \setminus v_1$	(6 5 2^2 1 0)	0.91
G_{333}	(6.41 6.24 5.41 3.59 2.59 1.76 0)	
$G_{333} \setminus v_1$	(5.30 4.86 3.25 1.70 0.89 0)	0.87
$G_{333} \setminus v_3$	(5.41 4.73 2.59 2 1.27 0)	0.49
G_{334}	(6.49 6 5.66 3.16 3 1.69 0)	
$G_{334} \setminus v_1$	(5.24 5 3 2 0.76 0)	0.93
$G_{334} \setminus v_3$	(5.41 4.73 2.59 2 1.27 0)	0.42
G_{335}	(7 6 5 3^2 2 0)	
$G_{335} \setminus v_1$	(6 5 2^2 1 0)	1
$G_{335} \setminus v_3$	(6 4 2^3 0)	0
G_{336}	(6.41 5.87 5.31 3.94 2.83 1.64 0)	
$G_{336} \setminus v_1$	(5.28 4.32 3 2.30 1.11 0)	0.53
$G_{336} \setminus v_2$	(5.12 4.55 3.47 2.14 0.73 0)	0.91
G_{337}	(6.34 6.11 4.86 4.37 2.62 1.70 0)	
$G_{337} \setminus v_1$	(5.12 4.55 3.47 2.14 0.73 0)	0.97
$G_{337} \setminus v_2$	(5.30 4 3.62 1.70 1.38 0)	0.32
G_{338}	(6.61 5.94 5 3.93 2.64 1.87 0)	
$G_{338} \setminus v_1$	(5.56 4 3 2 1.44 0)	0.43
$G_{338} \setminus v_2$	(5.30 4.86 3.25 1.70 0.89 0)	0.98
G_{339}	(6.58 6.10 4.60 4.21 2.69 1.81 0)	
$G_{339} \setminus v_1$	(5.30 4.86 3.25 1.70 0.89 0)	0.92
$G_{339} \setminus v_2$	(5.51 4 3.57 2 0.91 0)	0.90
G_{340}	(6.37 6 5.26 3.68 3.13 1.56 0)	
$G_{340} \setminus v_1$	(5.12 4.55 3.47 2.14 0.73 0)	0.83
$G_{340} \setminus v_2$	(5.28 4.32 3 2.30 1.11 0)	0.45
G_{341}	(6.37 6 5.26 3.68 3.13 1.56 0)	
$G_{341} \setminus v_1$	(5.28 4.32 3 2.30 1.11 0)	0.45
G_{342}	(6.53 5.91 5.35 3.29 3.12 1.80 0)	
$G_{342} \setminus v_1$	(5.37 4.53 3 2.21 0.89 0)	0.91

G_{343}	(6.57 6.15 4.70 3.89 3 1.70 0)	
$G_{343} \setminus v_1$	(5.56 4 3 2 1.44 0)	0.26
$G_{343} \setminus v_2$	(5.24 5 3 2 0.76 0)	0.94
G_{344}	(6.32 6 5 4 3.36 1.32 0)	
$G_{344} \setminus v_1$	(5.24 4 3^2 0.76 0)	0.56
$G_{344} \setminus v_2$	(5.09 4^2 2.43 0.49 0)	0.83
G_{345}	(7 5.41 5 4 2.59 2 0)	
$G_{345} \setminus v_1$	(6 4.41 3 1.59 1 0)	1
G_{346}	(6.39 6.30 4.77 4 2.70 1.84 0)	
$G_{346} \setminus v_1$	(5.37 4.53 3 2.21 0.89 0)	0.95
G_{347}	(6.39 6 4.77 4.62 2.38 1.84 0)	
$G_{347} \setminus v_1$	(5.30 4 3.62 1.70 1.38 0)	0.46
G_{348}	(6.60 6 5 3.66 3 1.74 0)	
$G_{348} \setminus v_1$	(5.51 4 3.57 2 0.91 0)	0.83
G_{349}	(6.32 6 5 4 3.36 1.32 0)	
$G_{349} \setminus v_1$	(5.24 4 3^2 0.76 0)	0.56
G_{350}	(6.27 5.81 5.27 3.92 2.91 1.82 0)	
$G_{350} \setminus v_1$	(5 4.41 3 2 1.59 0)	0.23
G_{351}	(6.56 5.60 4.75 4.31 3.11 1.67 0)	
$G_{351} \setminus v_1$	(5.12 4.55 3.47 2.14 0.73 0)	0.94
G_{352}	(6.70 5.51 5 3.88 3 1.90 0)	
$G_{352} \setminus v_1$	(5.51 4 3.57 2 0.91 0)	0.99
G_{353}	(6.63 5.81 4.63 4 3.05 1.88 0)	
$G_{353} \setminus v_1$	(5.37 4.53 3 2.21 0.89 0)	0.99
G_{354}	(6.18 5.70 5 4.67 2.79 1.66 0)	
$G_{354} \setminus v_1$	(4.73 4^2 2 1.27 0)	0.39
G_{355}	(6.41 6 4.41 4 3.59 1.59 0)	
$G_{355} \setminus v_1$	(5.24 4 3^2 0.76 0)	0.83
G_{356}	(6.49 5.66 5 4 3.16 1.69 0)	
$G_{356} \setminus v_1$	(5.24 4 3^2 0.76 0)	0.93
G_{357}	(6.15 6 4.83 4 3.56 1.45 0)	
$G_{357} \setminus v_1$	(5 4 3^2 1 0)	0.45
G_{358}	(6.60 6 4^2 3.66 1.74 0)	
$G_{358} \setminus v_1$	(5.56 3^3 1.44 0)	0.30
G_{359}	(6.49 5.66 5 4 3.16 1.69 0)	
$G_{359} \setminus v_1$	(5.24 4 3^2 0.76 0)	0.93
G_{360}	(6.56 5^2 4.68 3.32 1.44 0)	
$G_{360} \setminus v_1$	(5.09 4^2 2.43 0.49 0)	0.95
G_{361}	(7 6.24 4^2 3 1.76 0)	

$G_{361} \setminus v_1$	(6 4 3 2 1 0)	0.76
G_{362}	(6.48 5.93 4.90 3.91 2.68 2.11 0)	
$G_{362} \setminus v_1$	(5.30 4 3.62 1.70 1.38 0)	0.73
$G_{362} \setminus v_2$	(5.28 4.32 3 2.30 1.11 0)	1
G_{363}	(6.33 6.25 4.55 3.80 3.20 1.87 0)	
$G_{363} \setminus v_1$	(5.28 4.32 3 2.30 1.11 0)	0.76
G_{364}	(6.64 6 4.72 3.41 3 2.22 0)	
$G_{364} \setminus v_1$	(5.41 4.73 2.59 2 1.27 0)	0.95
$G_{364} \setminus v_2$	(5.56 4 3 2 1.44 0)	0.78
G_{365}	(6.64 6 4.72 3.41 3 2.22 0)	
$G_{365} \setminus v_1$	(5.56 4 3 2 1.44 0)	0.78
G_{366}	(6.41 6 4.41 4 3.59 1.59 0)	
$G_{366} \setminus v_1$	(5.24 4 3^2 0.76 0)	0.83
G_{367}	(7 5^2 4 3 2 0)	
$G_{367} \setminus v_1$	(6 4 3 2 1 0)	1
G_{368}	(7 6 4 3^3 0)	
$G_{368} \setminus v_1$	(6 4 2^3 0)	1
G_{369}	(6.61 5.49 4.80 3.77 3.26 2.07 0)	
$G_{369} \setminus v_1$	(5.28 4.32 3 2.30 1.11 0)	0.96
G_{370}	(6.35 5.91 4.76 3.85 2.80 2.33 0)	
$G_{370} \setminus v_1$	(5 4.41 3 2 1.59 0)	0.74
G_{371}	(6.39 5.62 4.77 4 3.38 1.84 0)	
$G_{371} \setminus v_1$	(5 4 3^2 1 0)	0.84
G_{372}	(6.72 5.67 4.48 3.71 3 2.42 0)	
$G_{372} \setminus v_1$	(5.56 4 3 2 1.44 0)	0.98
G_{373}	(6.62 5.41 4.62 4.38 2.59 2.38 0)	
$G_{373} \setminus v_1$	(5.30 4 3.62 1.70 1.38 0)	1
G_{374}	(6.66 5.85 4^2 3.15 2.34 0)	
$G_{374} \setminus v_1$	(5.56 3^3 1.44 0)	0.90
G_{375}	(6.41 5.41^2 3.59 2.59^2 0)	
$G_{375} \setminus v_1$	(5 4.41 3 2 1.59 0)	1
G_{376}	(6.41 5.41^2 3.59 2.59^2 0)	
$G_{376} \setminus v_1$	(5 4.41 3 2 1.59 0)	1
G_{377}	(6.32 5.58 5 4.13 3 1.97 0)	
$G_{377} \setminus v_1$	(4.73 4^2 2 1.27 0)	0.70
G_{378}	(6.34 5.41 5 4.47 2.59 2.19 0)	
$G_{378} \setminus v_1$	(4.73 4^2 2 1.27 0)	0.92
G_{379}	(6.60 5^2 4 3.66 1.74 0)	
$G_{379} \setminus v_1$	(5.24 4 3^2 0.76 0)	0.98
G_{380}	(7 6^2 5 3 1 0)	

$G_{380} \setminus v_1$	(6 5 4 2 1 0)	0
G_{381}	(6.40 6^2 5.48 3.17 0.94 0)	
$G_{381} \setminus v_1$	(5^3 3 0^2)	0.94
$G_{381} \setminus v_4$	(5.37 5 4.53 2.21 0.89 0)	0.05
G_{382}	(7 6^2 4^2 1 0)	
$G_{382} \setminus v_1$	(6 5 3^2 1 0)	0
G_{383}	(6.53 6^2 4.52 4 0.95 0)	
$G_{383} \setminus v_1$	(5^3 3 0^2)	0.95
$G_{383} \setminus v_3$	(5.51 5 3.57 3 0.91 0)	0.04
G_{384}	(6.27 6^2 5 3.86 0.87 0)	
$G_{384} \setminus v_1$	(5.24 5 4 3 0.76 0)	0.11
G_{385}	(7 6.24 5.73 5 2.27 1.76 0)	
$G_{385} \setminus v_1$	(6 5 4 2 1 0)	0.76
G_{386}	(6.45 6^2 5.41 2.59 1.55 0)	
$G_{386} \setminus v_1$	(5.37 5 4.53 2.21 0.89 0)	0.66
G_{387}	(7^2 6 3^2 2 0)	
$G_{387} \setminus v_1$	(6^2 2^3 0)	0
G_{388}	(7 6.30 5.86 4.25 2.70 1.89 0)	
$G_{388} \setminus v_1$	(6 5.17 3.31 2 1.52 0)	0.37
$G_{388} \setminus v_2$	(6 5 4 2 1 0)	0.89
G_{389}	(7 6.24 6 4 3 1.76 0)	
$G_{389} \setminus v_1$	(6 5.17 3.31 2 1.52 0)	0.24
G_{390}	(6.39 6.30 6 4.77 2.70 1.84 0)	
$G_{390} \setminus v_1$	(5.30^2 4 1.70^2 0)	0.14
$G_{390} \setminus v_4$	(5.37 5 4.53 2.21 0.89 0)	0.95
G_{391}	(7 6.12 5.55 4.47 3.14 1.73 0)	
$G_{391} \setminus v_1$	(6 4.62 3.62 2.38 1.38 0)	0.35
G_{392}	(7 6.51 5 4.57 3 1.91 0)	
$G_{392} \setminus v_1$	(6 5 4 2 1 0)	0.91
G_{393}	(7 6.37 5.53 4 3.21 1.89 0)	
$G_{393} \setminus v_1$	(6 5 3^2 1 0)	0.89
G_{394}	(7 6.09 5^2 3.43 1.49 0)	
$G_{394} \setminus v_1$	(6 4^2 3 1 0)	0.44
G_{395}	(6.59 6.25 5.59 4.55 3.20 1.82 0)	
$G_{395} \setminus v_1$	(5.37 5 4.53 2.21 0.89 0)	0.93
$G_{395} \setminus v_2$	(5.51 5 3.57 3 0.91 0)	0.91
$G_{395} \setminus v_3$	(5.59 4.86 3.64 2.30 1.61 0)	0.21
G_{396}	(6.63 6 5.81 4.63 3.05 1.88 0)	
$G_{396} \setminus v_1$	(5.37 5 4.53 2.21 0.89 0)	0.99
$G_{396} \setminus v_3$	(5.59 4.86 3.64 2.30 1.61 0)	0.27

G_{397}	(6.41 6^2 4.41 3.59 1.59 0)	
$G_{397} \setminus v_1$	(5.24 5 4 3 0.76 0)	0.83
$G_{397} \setminus v_3$	(5.34 5 3.47 3 1.19 0)	0.40
G_{398}	(7 6 5^2 3 2 0)	
$G_{398} \setminus v_1$	(6 5 4 2 1 0)	1
$G_{398} \setminus v_3$	(6 4^2 2^2 0)	0
G_{399}	(6.49 5.66 5 4 3.16 1.69 0)	
$G_{399} \setminus v_1$	(5.24 5 4 3 0.76 0)	0.93
$G_{399} \setminus v_3$	(5.41 4.73 4 2.59 1.27 0)	0.42
G_{400}	(6.60 6^2 4 3.66 1.74 0)	
$G_{400} \setminus v_1$	(5.56 5 3^2 1.44 0)	0.30
$G_{400} \setminus v_3$	(5.51 5 3.57 3 0.91 0)	0.83
G_{401}	(6.60 6^2 4 3.66 1.74 0)	
$G_{401} \setminus v_1$	(5.56 5 3^2 1.44 0)	0.30
G_{402}	(6.57 6.15 5 4.70 3.89 1.70 0)	
$G_{402} \setminus v_1$	(5.56 4^2 3 1.44 0)	0.26
$G_{402} \setminus v_2$	(5.24 5 4 3 0.76 0)	0.94
G_{403}	(6.70 6 5.51 4 3.88 1.90 0)	
$G_{403} \setminus v_1$	(5.51 5 3.57 3 0.91 0)	0.99
$G_{403} \setminus v_2$	(5.68 4.54 3^2 1.79 0)	0.11
G_{404}	(6.62 6.39 4.77 4.38 4 1.84 0)	
$G_{404} \setminus v_1$	(5.51 5 3.57 3 0.91 0)	0.93
G_{405}	(6.39 6 5.62 4.77 3.38 1.84 0)	
$G_{405} \setminus v_1$	(5.30 4.62 4 2.38 1.70 0)	0.14
G_{406}	(7 6 5 4^2 2 0)	
$G_{406} \setminus v_1$	(6 5 3^2 1 0)	1
G_{407}	(7^2 5 3^3 0)	
$G_{407} \setminus v_1$	(6^2 2^3 0)	1
G_{408}	(7 6.28 5.32 4 3.30 2.11 0)	
$G_{408} \setminus v_1$	(6 4.62 3.62 2.38 1.38 0)	0.73
G_{409}	(7 6.30 5 4.62 2.70 2.38 0)	
$G_{409} \setminus v_1$	(6 4.62 3.62 2.38 1.38 0)	1
G_{410}	(7 6.56 5 4 3 2.44 0)	
$G_{410} \setminus v_1$	(6 5.17 3.31 2 1.52 0)	0.92
G_{411}	(7 6.24 5 4^2 1.76 0)	
$G_{411} \setminus v_1$	(6 4^2 3 1 0)	0.76
G_{412}	(6.41 6.32 5.58 4.13 3.59 1.97 0)	
$G_{412} \setminus v_1$	(5.34 5 3.47 3 1.19 0)	0.78
$G_{412} \setminus v_3$	(5.41 4.73 4 2.59 1.27 0)	0.70
G_{413}	(6.62 6.30 5.41 4.38 2.70 2.59 0)	

$G_{413} \setminus v_1$	(5.59 4.86 3.64 2.30 1.61 0)	0.98
$G_{413} \setminus v_3$	(5.30^2 4 1.70^2 0)	0.89
G_{414}	(6.66 6 5.85 4 3.15 2.34 0)	
$G_{414} \setminus v_1$	(5.56 5 3^2 1.44 0)	0.90
$G_{414} \setminus v_3$	(5.59 4.86 3.64 2.30 1.61 0)	0.73
G_{415}	(7 6.17 5 4.31 3 2.52 0)	
$G_{415} \setminus v_1$	(6 4^2 2^2 0)	0.52
$G_{415} \setminus v_3$	(6 5.17 3.31 2 1.52 0)	1
G_{416}	(6.73 6 5.41 4 3.27 2.59 0)	
$G_{416} \setminus v_1$	(5.59 4.86 3.64 2.30 1.61 0)	0.98
$G_{416} \setminus v_2$	(5.58 4.54 3^2 1.79 0)	0.80
G_{417}	(6.62 6.11 5 4.38 3.75 2.14 0)	
$G_{417} \setminus v_1$	(5.34 5 3.47 3 1.19 0)	0.95
$G_{417} \setminus v_2$	(5.56 4^2 3 1.44 0)	0.70
G_{418}	(6.70 6.25 4.76 4.55 3.20 2.54 0)	
$G_{418} \setminus v_1$	(5.59 4.86 3.64 2.30 1.61 0)	0.93
G_{419}	(6.68 6.32 4.86 4 3.77 2.37 0)	
$G_{419} \setminus v_1$	(5.56 5 3^2 1.44 0)	0.93
$G_{419} \setminus v_2$	(5.68 4.54 3^2 1.79 0)	0.58
G_{420}	(6.64 6 5 4.72 3.41 2.22 0)	
$G_{420} \setminus v_1$	(5.41 4.73 4 2.59 1.27 0)	0.95
$G_{420} \setminus v_2$	(5.56 4^2 3 1.44 0)	0.78
G_{421}	(6.48 6 5.41 4.69 2.83 2.59 0)	
$G_{421} \setminus v_1$	(5.30 4.62 4 2.38 1.70 0)	0.89
G_{422}	(7 5.62 5 4.62 3.38 2.38 0)	
$G_{422} \setminus v_1$	(6 4.62 3.62 2.38 1.38 0)	1
G_{423}	(6.73 6 5.41 4 3.27 2.59 0)	
$G_{423} \setminus v_1$	(5.68 4.54 3^2 1.79 0)	0.80
G_{424}	(6.64 6 5 4.72 3.41 2.22 0)	
$G_{424} \setminus v_1$	(5.56 4^2 3 1.44 0)	0.78
G_{425}	(7 6 5 4 3^2 0)	
$G_{425} \setminus v_1$	(6 4^2 2^2 0)	1
G_{426}	(7 5^3 4 2 0)	
$G_{426} \setminus v_1$	(6 4^2 3 1 0)	1
G_{427}	(6.77 6 4.81 4 3.64 2.77 0)	
$G_{427} \setminus v_1$	(5.68 4.54 3^2 1.79 0)	0.98
G_{428}	(6.72 5.67 5 4.48 3.71 2.42 0)	
$G_{428} \setminus v_1$	(5.56 4^2 3 1.44 0)	0.98
G_{429}	(6.62 5.62 5.41 4.38 3.38 2.59 0)	
$G_{429} \setminus v_1$	(5.30 4.62 4 2.38 1.70 0)	0.89

G_{430}	(6.41 6 5.41 4 3.59 2.59 0)	
$G_{430} \setminus v_1$	(5^2 3^2 2 0)	0.59
G_{431}	(7 6 4^3 3 0)	
$G_{431} \setminus v_1$	(6 3^4 0)	0
G_{432}	(7 6^3 4 1 0)	
$G_{432} \setminus v_1$	(6 5^2 3 1 0)	0
G_{433}	(6.53 6^3 4.52 0.95 0)	
$G_{433} \setminus v_1$	(5.51 5^2 3.57 0.91 0)	0.04
$G_{433} \setminus v_5$	(5^4 0^2)	0.95
G_{434}	(7^2 6 5 3 2 0)	
$G_{434} \setminus v_1$	(6^2 4 2^2 0)	0
G_{435}	(7 6.37 6 5.53 3.21 1.89 0)	
$G_{435} \setminus v_1$	(6 5^2 3 1 0)	0.89
$G_{435} \setminus v_3$	(6 5.30 4.62 2.38 1.70 0)	0.19
G_{436}	(6.60 6^3 3.66 1.74 0)	
$G_{436} \setminus v_1$	(5.56 5^2 3 1.44 0)	0.30
$G_{436} \setminus v_5$	(5.51 5^2 3.57 0.91 0)	0.83
G_{437}	(7 6.51 6 4.57 4 1.91 0)	
$G_{437} \setminus v_1$	(6 5^2 3 1 0)	0.91
$G_{437} \setminus v_2$	(6 5.48 3.69 3 1.83 0)	0.08
G_{438}	(7 6.24 6 5 4 1.76 0)	
$G_{438} \setminus v_1$	(6 5.17 4 3.31 1.52 0)	0.24
G_{439}	(6.62 6.39 6 4.77 4.38 1.84 0)	
$G_{439} \setminus v_1$	(5.51 5^2 3.57 0.91 0)	0.93
$G_{439} \setminus v_4$	(5.62 5.30 4 3.38 1.70 0)	0.14
G_{440}	(6.70 6^2 5.51 3.88 1.90 0)	
$G_{440} \setminus v_1$	(5.51 5^2 3.57 0.91 0)	0.99
$G_{440} \setminus v_4$	(5.68 5 4.54 3 1.79 0)	0.11
G_{441}	(7 6^2 5 4 2 0)	
$G_{441} \setminus v_1$	(6 5^2 3 1 0)	1
$G_{441} \setminus v_4$	(6 5 4 3 2 0)	0
G_{442}	(7^2 5 3^3 0)	
$G_{442} \setminus v_1$	(6^2 2^3 0)	1
G_{443}	(7^2 6.17 4.31 3 2.52 0)	
$G_{443} \setminus v_1$	(6^2 4 2^2 0)	0.52
G_{444}	(7 6.30^2 5 2.70^2 0)	
$G_{444} \setminus v_1$	(6 5.30 4.62 2.38 1.70 0)	1
G_{445}	(7 6.59 5.86 4.64 3.30 2.61 0)	
$G_{445} \setminus v_1$	(6 5.48 3.69 3 1.83 0)	0.78
$G_{445} \setminus v_2$	(6 5.30 4.62 2.38 1.70 0)	0.91

G_{446}	(7 6.34 6 4.47 4 2.19 0)	
$G_{446} \setminus v_1$	(6 5.17 4 3.31 1.52 0)	0.67
G_{447}	(7 6.56 6 4^2 2.44 0)	
$G_{447} \setminus v_1$	(6 5.48 3.69 3 1.83 0)	0.61
G_{448}	(7 6.41 5.73 5 3.59 2.27 0)	
$G_{448} \setminus v_1$	(6 5.17 4 3.31 1.52 0)	0.75
G_{449}	(7^2 5^2 3^2 0)	
$G_{449} \setminus v_1$	(6^2 4 2^2 0)	1
G_{450}	(6.68 6.32 6 4.86 3.77 2.37 0)	
$G_{450} \setminus v_1$	(5.68 5 4.54 3 1.79 0)	0.58
$G_{450} \setminus v_2$	(5.62 5.30 4 3.38 1.70 0)	0.67
$G_{450} \setminus v_4$	(5.56 5^2 3 1.44 0)	0.93
G_{451}	(7 6.30 5.62 5 3.38 2.70 0)	
$G_{451} \setminus v_1$	(6 5 4 3 2 0)	0.70
$G_{451} \setminus v_4$	(6 5.30 4.62 2.38 1.70 0)	1
G_{452}	(6.73 6^2 5.41 3.27 2.59 0)	
$G_{452} \setminus v_1$	(5.68 5 4.54 3 1.79 0)	0.80
G_{453}	(7 6.68 5.54 4^2 2.79 0)	
$G_{453} \setminus v_1$	(6 5.48 3.69 3 1.83 0)	0.96
G_{454}	(7 6.56 5^2 4 2.44 0)	
$G_{454} \setminus v_1$	(6 5.17 4 3.31 1.52 0)	0.92
G_{455}	(7 6.30 5.62 5 3.38 2.70 0)	
$G_{455} \setminus v_1$	(6 4.62^2 2.38^2 0)	0.32
G_{456}	(6.74 6.37 5.59 4.73 3.90 2.67 0)	
$G_{456} \setminus v_1$	(5.62 5.30 4 3.38 1.70 0)	0.97
$G_{456} \setminus v_2$	(5.68 5 4.54 3 1.79 0)	0.88
$G_{456} \setminus v_3$	(5.73 5 4 3 2.27 0)	0.40
G_{457}	(6.77 6^2 4.81 3.64 2.77 0)	
$G_{457} \setminus v_1$	(5.68 5 4.54 3 1.79 0)	0.98
$G_{457} \setminus v_3$	(5.73 5 4 3 2.27 0)	0.50
G_{458}	(7 6.48 5 4.69 4 2.83 0)	
$G_{458} \setminus v_1$	(6 5 4 3 2 0)	0.83
$G_{458} \setminus v_3$	(6 5.48 3.69 3 1.83 0)	1
G_{459}	(7 6^2 4^2 3 0)	
$G_{459} \setminus v_1$	(6 5 3^3 0)	0
$G_{459} \setminus v_3$	(6 5 4 3 2 0)	1
G_{460}	(7 6.17 5^2 4.31 2.52 0)	
$G_{460} \setminus v_1$	(6 5.17 4 3.31 1.52 0)	1
$G_{460} \setminus v_3$	(6 4^3 2 0)	0.52
G_{461}	(6.62^2 5.41 4.38^2 2.59 0)	

$G_{461} \setminus v_1$	(5.62 5.30 4 3.38 1.70 0)	0.89
G_{462}	(7 6^2 4^2 3 0)	
$G_{462} \setminus v_1$	(6 5 3^3 0)	0
G_{463}	(6.80 6.25 5.45 4.55 3.75 3.20 0)	
$G_{463} \setminus v_1$	(5.73 5 4 3 2.27 0)	0.93
G_{464}	(7 6.41 5 4^2 3.59 0)	
$G_{464} \setminus v_1$	(6 5 3^3 0)	0.59
G_{465}	(7 6 5^2 4 3 0)	
$G_{465} \setminus v_1$	(6 4^3 2 0)	1
G_{466}	(7 5.62^2 5 3.38^2 0)	
$G_{466} \setminus v_1$	(6 4.62^2 2.38^2 0)	1
G_{467}	(7 6^4 1 0)	
$G_{467} \setminus v_1$	(6 5^3 1 0)	0
G_{468}	(7^2 6^2 4 2 0)	
$G_{468} \setminus v_1$	(6^2 5 3 2 0)	0
G_{469}	(7 6.51 6^2 4.57 1.91 0)	
$G_{469} \setminus v_1$	(6 5^3 1 0)	0.91
$G_{469} \setminus v_4$	(6 5.48 5 3.69 1.83 0)	0.08
G_{470}	(7 6^3 5 2 0)	
$G_{470} \setminus v_1$	(6 5^3 1 0)	1
$G_{470} \setminus v_6$	(6 5^2 4 2 0)	0
G_{471}	(7^3 5 3^2 0)	
$G_{471} \setminus v_1$	(6^2 4 2^2 0)	1
G_{472}	(7^2 6.30 5.62 3.38 2.70 0)	
$G_{472} \setminus v_1$	(6^2 5 3 2 0)	0.70
G_{473}	(7 6.56 6^2 4 2.44 0)	
$G_{473} \setminus v_1$	(6 5.48 5 3.69 1.83 0)	0.61
G_{474}	(7^2 6.48 4.69 4 2.83 0)	
$G_{474} \setminus v_1$	(6^2 5 3 2 0)	0.83
G_{475}	(7^2 6.17 5 4.31 2.52 0)	
$G_{475} \setminus v_1$	(6^2 4^2 2 0)	0.52
G_{476}	(7 6.62 6.30 5 4.38 2.70 0)	
$G_{476} \setminus v_1$	(6 5.62 4.62 3.38 2.38 0)	0.32
$G_{476} \setminus v_3$	(6 5.48 5 3.69 1.83 0)	0.87
G_{477}	(7 6.68 6 5.54 4 2.79 0)	
$G_{477} \setminus v_1$	(6 5.48 5 3.69 1.83 0)	0.96
$G_{477} \setminus v_3$	(6 5.62 4.62 3.38 2.38 0)	0.41
G_{478}	(7^2 6 5 4 3 0)	
$G_{478} \setminus v_1$	(6^2 5 3 2 0)	1
$G_{478} \setminus v_3$	(6^2 4 3^2 0)	0

G_{479}	(7 6.48 6 5 4.69 2.83 0)	
$G_{479} \setminus v_1$	(6 5.48 5 3.69 1.83 0)	1
$G_{479} \setminus v_3$	(6 5^2 4 2 0)	0.83
$G_{479} \setminus v_5$	(6 5.41 4^2 2.59 0)	0.24
G_{480}	(7 6^3 4 3 0)	
$G_{480} \setminus v_1$	(6 5^2 4 2 0)	1
$G_{480} \setminus v_5$	(6 5^2 3^2 0)	0
G_{481}	(7 5.62^2 5 3.38^2 0)	
$G_{481} \setminus v_1$	(6 4.62^2 2.38^2 0)	1
G_{482}	(7 6.73 6 5 4 3.27 0)	
$G_{482} \setminus v_1$	(6 5.62 4.62 3.38 2.38 0)	0.89
G_{483}	(7^2 6 4^3 0)	
$G_{483} \setminus v_1$	(6^2 4 3^2 0)	1
G_{484}	(7^2 5^3 3 0)	
$G_{484} \setminus v_1$	(6^2 4^2 2 0)	1
G_{485}	(7 6.62 5.62 5 4.38 3.38 0)	
$G_{485} \setminus v_1$	(6 5.62 4.62 3.38 2.38 0)	1
$G_{485} \setminus v_4$	(6 5.41 4^2 2.59 0)	0.79
G_{486}	(7 6.41 6 5 4 3.59 0)	
$G_{486} \setminus v_1$	(6 5^2 3^2 0)	0.59
$G_{486} \setminus v_4$	(6 5.41 4^2 2.59 0)	1
G_{487}	(7^2 5^2 4^2 0)	
$G_{487} \setminus v_1$	(6^2 4 3^2 0)	1
G_{488}	(7^2 6^3 2 0)	
$G_{488} \setminus v_1$	(6^2 5^2 2 0)	0
G_{489}	(7^3 6 4 3 0)	
$G_{489} \setminus v_1$	(6^3 3^2 0)	0
G_{490}	(7^2 6.48 6 4.69 2.83 0)	
$G_{490} \setminus v_1$	(6^2 5^2 2 0)	0.83
$G_{490} \setminus v_3$	(6^2 5.41 4 2.59 0)	0.24
G_{491}	(7^2 6^2 5 3 0)	
$G_{491} \setminus v_1$	(6^2 5^2 2 0)	1
$G_{491} \setminus v_5$	(6^2 5 4 3 0)	0
G_{492}	(7^4 5^2 0)	
$G_{492} \setminus v_1$	(6^4 4 0)	1
G_{493}	(7^2 6.62 5.62 4.38 3.38 0)	
$G_{493} \setminus v_1$	(6^2 5.41 4 2.59 0)	0.79
G_{494}	(7^3 5 4^2 0)	
$G_{494} \setminus v_1$	(6^3 3^2 0)	1
G_{495}	(7^2 6.41 5^2 3.59 0)	

$G_{495} \setminus v_1$	$(6^2\ 5.41\ 4\ 2.59\ 0)$	1
$G_{495} \setminus v_3$	$(6^2\ 5\ 4\ 3\ 0)$	0.59
G_{496}	$(7^2\ 6\ 5^2\ 4\ 0)$	
$G_{496} \setminus v_1$	$(6^2\ 5\ 4\ 3\ 0)$	1
$G_{496} \setminus v_3$	$(6^2\ 4^3\ 0)$	0
G_{497}	$(7^3\ 6^2\ 3\ 0)$	
$G_{497} \setminus v_1$	$(6^3\ 5\ 3\ 0)$	0
G_{498}	$(7^4\ 6\ 4\ 0)$	
$G_{498} \setminus v_1$	$(6^4\ 4\ 0)$	0
G_{499}	$(7^3\ 6.41\ 5\ 3.59\ 0)$	
$G_{499} \setminus v_1$	$(6^3\ 5\ 3\ 0)$	0.59
G_{500}	$(7^3\ 6\ 5\ 4\ 0)$	
$G_{500} \setminus v_1$	$(6^3\ 5\ 3\ 0)$	1
$G_{500} \setminus v_4$	$(6^3\ 4^2\ 0)$	0
G_{501}	$(7^3\ 5^3\ 0)$	
$G_{501} \setminus v_1$	$(6^3\ 4^2\ 0)$	1
G_{502}	$(7^4\ 6\ 4\ 0)$	
$G_{502} \setminus v_1$	$(6^4\ 4\ 0)$	0
G_{503}	$(7^4\ 5^2\ 0)$	
$G_{503} \setminus v_1$	$(6^4\ 4\ 0)$	1
G_{504}	$(7^5\ 5\ 0)$	
$G_{504} \setminus v_1$	$(6^5\ 0)$	-1
G_{505}	$(7\ 6\ 2^3\ 1\ 0)$	
$G_{505} \setminus v_1$	$(6\ 1^4\ 0)$	0